Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.2-Cosine/95-4.2.7-d-trig-^m-a+b-c-cos-^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [98]. This is test number [95].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (98)	0.00 (0)
Mathematica	100.00 (98)	0.00 (0)
Maple	97.96 (96)	2.04 (2)
Fricas	86.73 (85)	13.27 (13)
Giac	77.55 (76)	22.45 (22)
Maxima	71.43 (70)	28.57 (28)
Mupad	68.37 (67)	31.63 (31)
Sympy	19.39 (19)	80.61 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

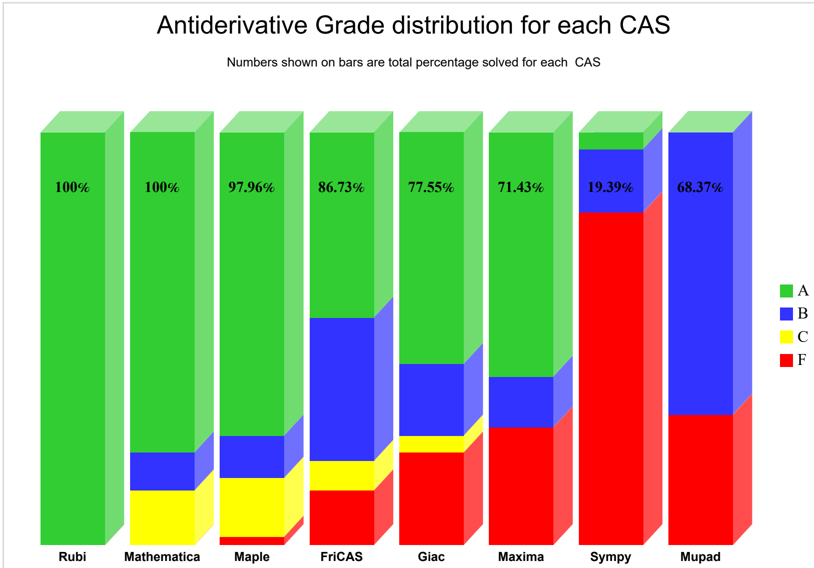
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

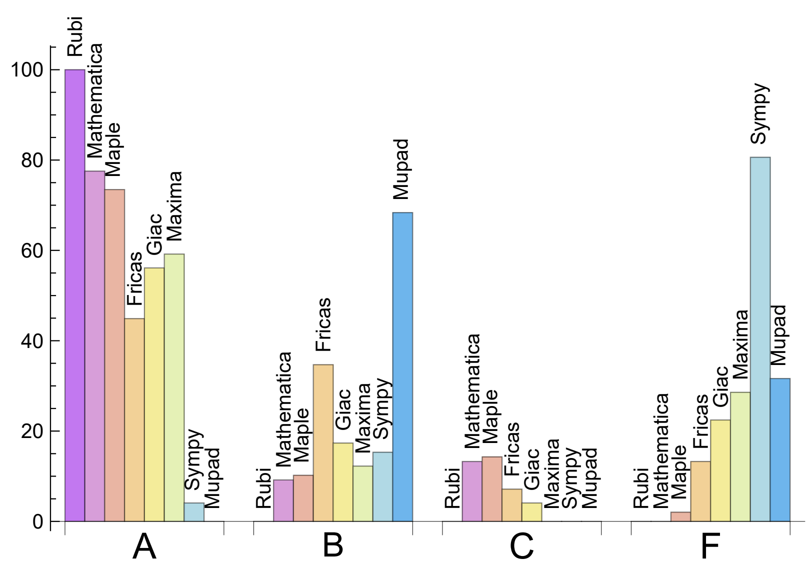
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	77.551	9.184	13.265	0.000
Maple	73.469	10.204	14.286	2.041
Maxima	59.184	12.245	0.000	28.571
Giac	56.122	17.347	4.082	22.449
Fricas	44.898	34.694	7.143	13.265
Sympy	4.082	15.306	0.000	80.612
Mupad	0.000	68.367	0.000	31.633

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Fricas	13	61.54	0.00	38.46
Giac	22	90.91	9.09	0.00
Maxima	28	100.00	0.00	0.00
Mupad	31	0.00	100.00	0.00
Sympy	79	67.09	32.91	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.32
Maxima	0.33
Giac	0.34
Fricas	0.45
Mathematica	0.88
Maple	0.89
Mupad	2.66
Sympy	6.61

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	54.61	1.16	45.50	0.95
Maxima	58.53	1.49	41.50	1.09
Mathematica	67.14	1.14	45.00	1.00
Giac	68.01	1.43	46.50	1.24
Rubi	73.12	1.01	41.00	1.00
Mupad	274.70	3.00	75.00	1.06
Sympy	4203.79	142.58	78.00	3.50
Fricas	15965.06	72.79	123.00	2.90

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

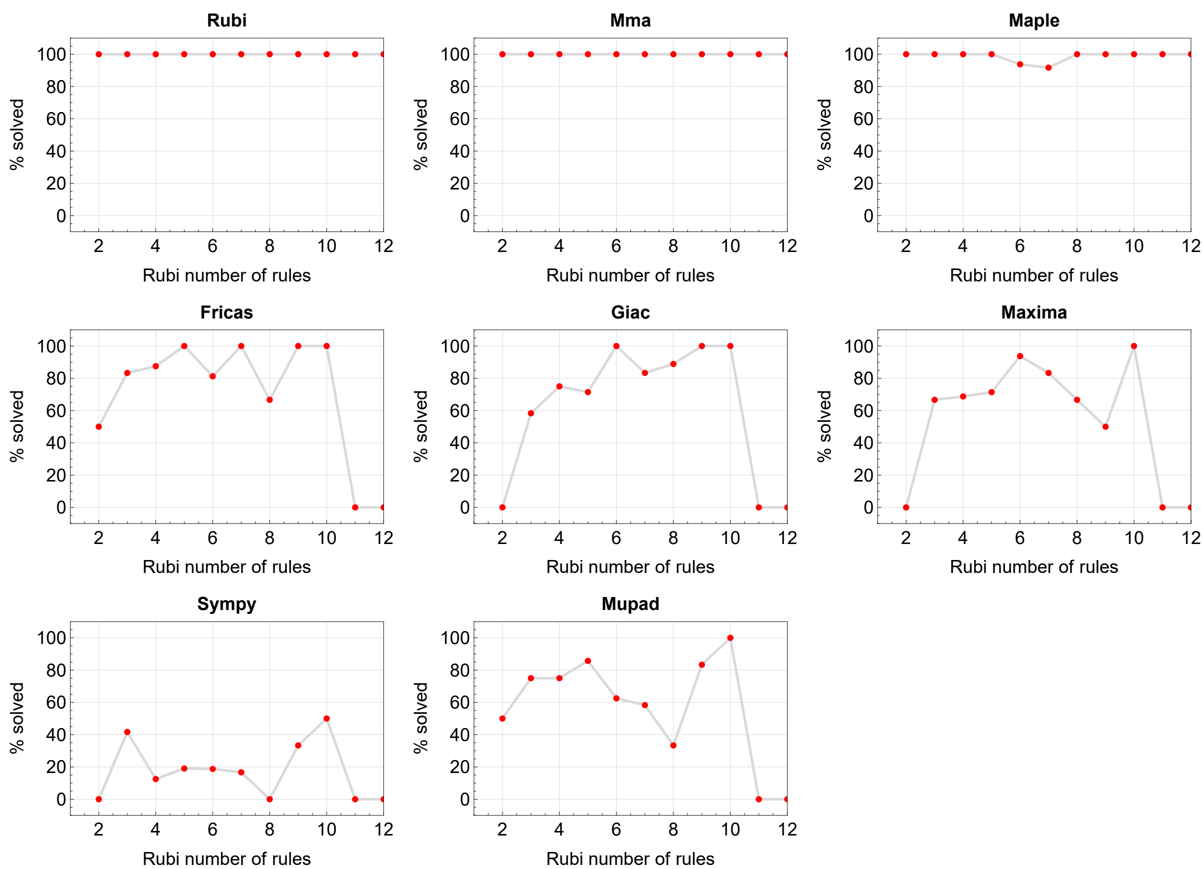


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

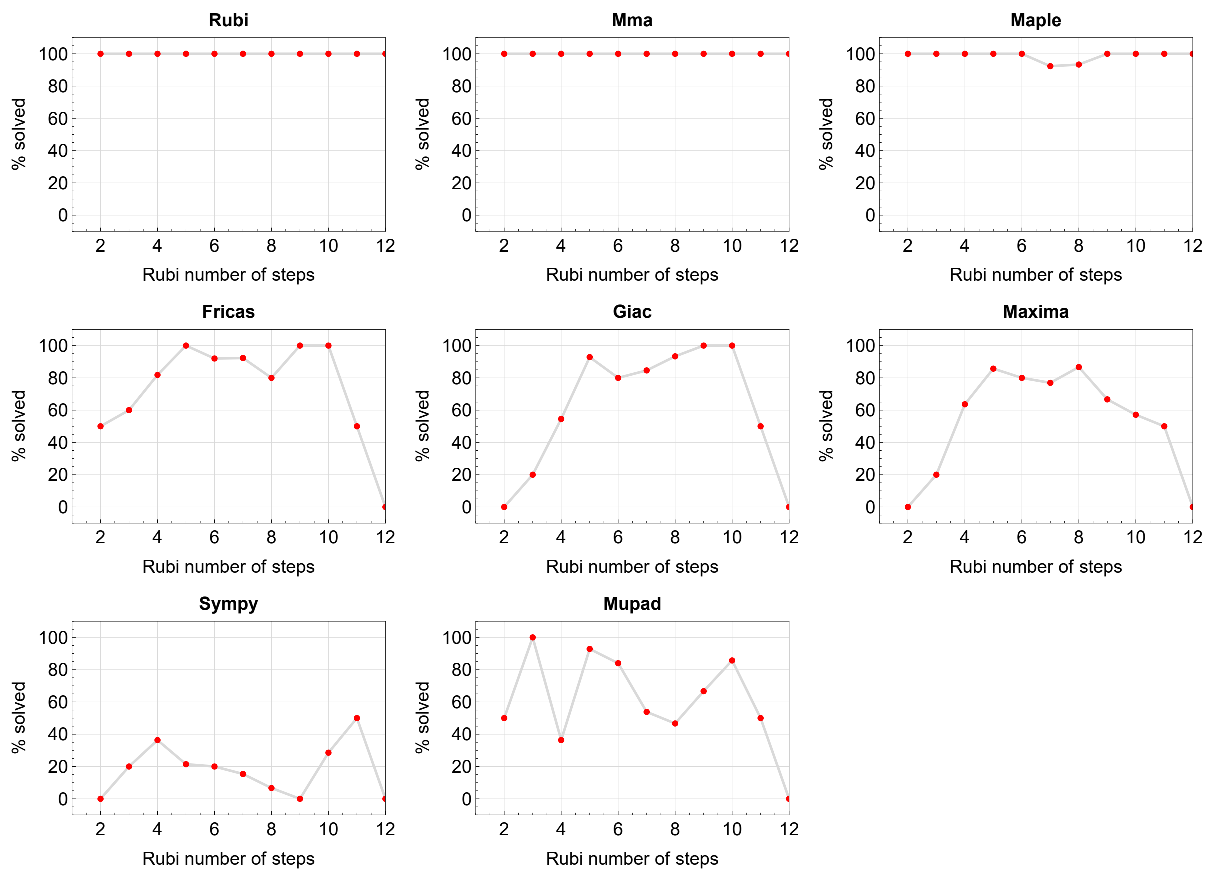


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

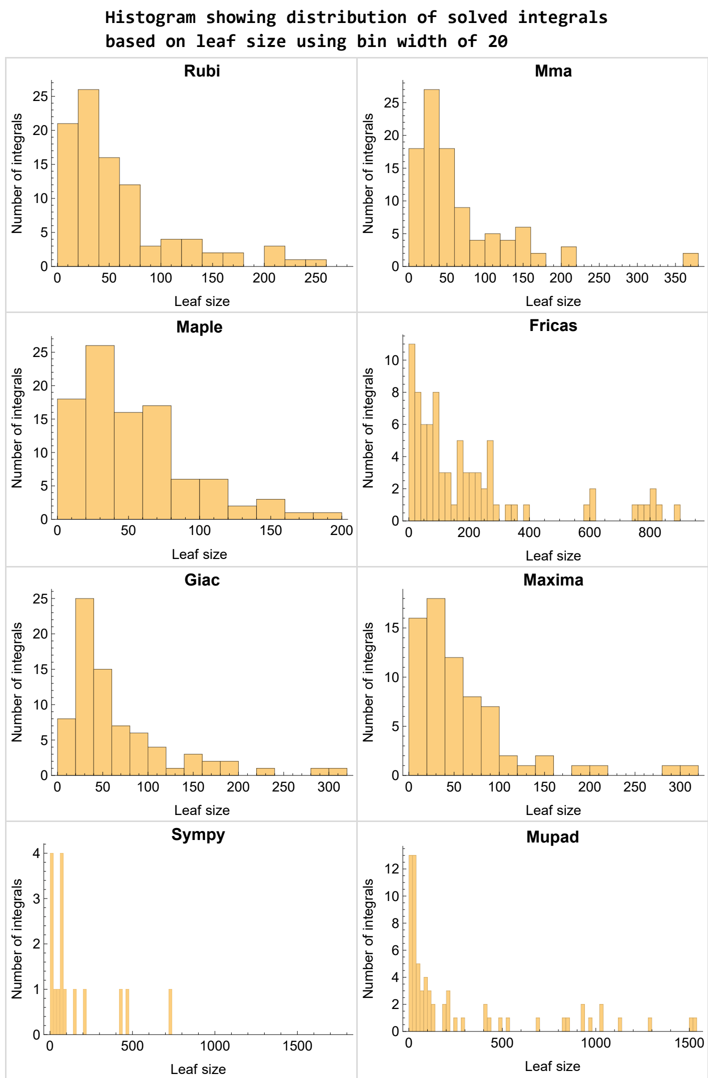


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

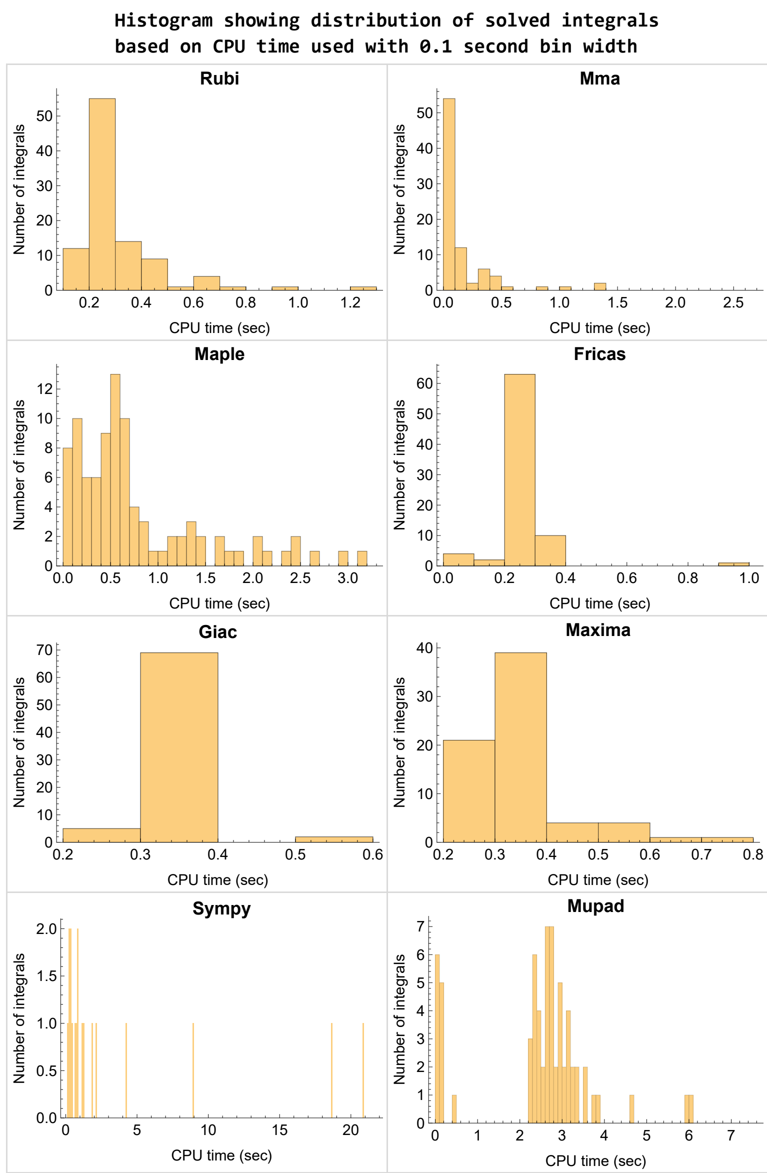


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

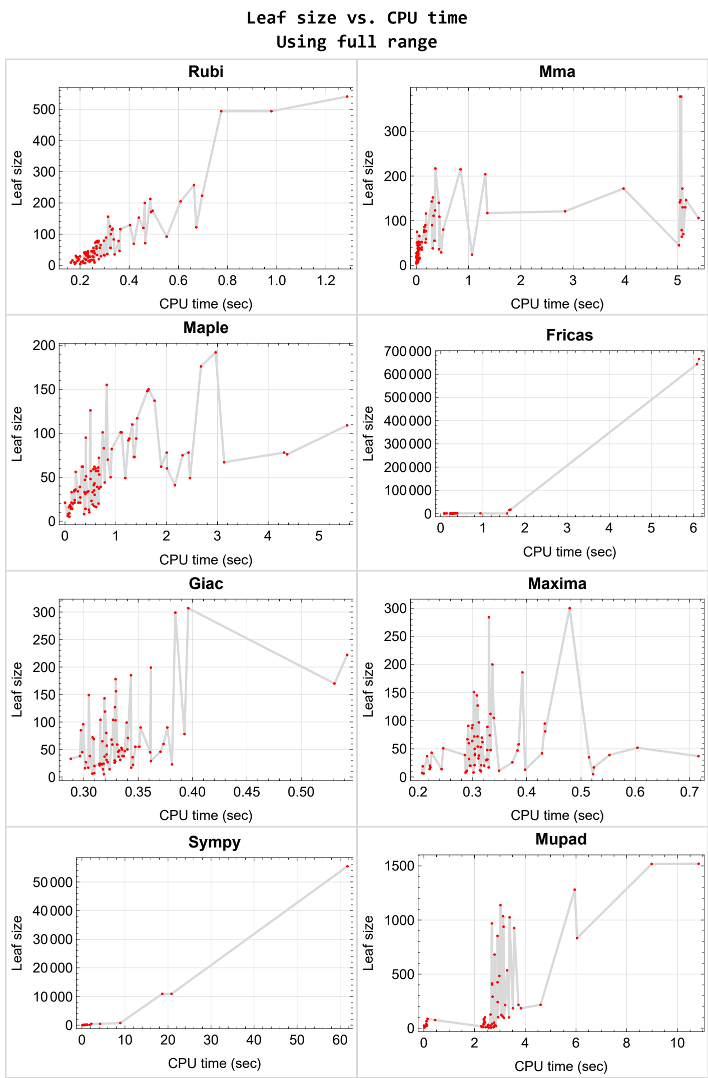


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{74, 76, 77, 79\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

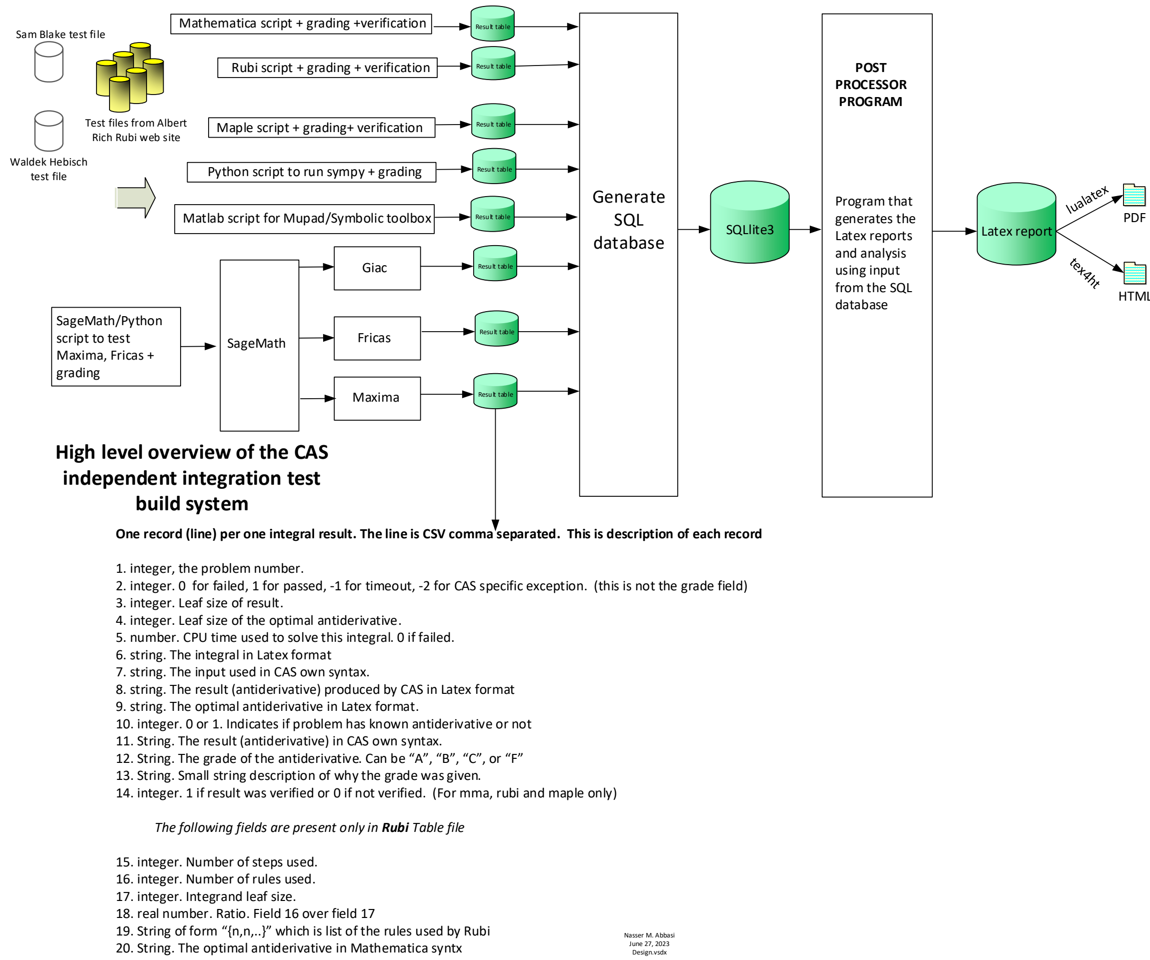
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 8, 10, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 73, 81, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98 }

B grade { 6, 7, 9, 11, 12, 15, 16, 33, 34 }

C grade { 70, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 92 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 57, 59, 60, 62, 63, 65, 66, 71, 73, 81, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98 }

B grade { 52, 55, 56, 58, 61, 64, 67, 68, 69, 85 }

C grade { 24, 70, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 92 }

F normal fail { 93, 94 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 17, 18, 19, 24, 25, 28, 29, 30, 32, 33, 34, 35, 36, 37, 44, 45, 46, 47, 49, 51, 53, 62, 73, 86, 87, 88, 89, 90, 93, 95, 96, 97, 98 }

B grade { 6, 7, 9, 15, 16, 20, 21, 22, 23, 26, 27, 31, 38, 39, 40, 41, 42, 43, 61, 64, 65, 67, 68, 69, 70, 71, 76, 79, 80, 81, 82, 83, 85, 91 }

C grade { 63, 66, 72, 75, 78, 84, 92 }

F normal fail { 48, 50, 55, 56, 57, 58, 59, 60 }

F(-1) timedout fail { }

F(-2) exception fail { 52, 54, 74, 77, 94 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 67, 68, 69, 73, 81, 86, 92, 93, 94, 95, 97, 98 }

B grade { 6, 7, 15, 16, 51, 53, 54, 87, 88, 89, 90, 91 }

C grade { }

F normal fail { 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 96 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 70, 72, 73, 86, 87, 89, 92, 93, 94, 95, 96, 97, 98 }

B grade { 6, 7, 15, 16, 23, 47, 49, 53, 67, 69, 71, 81, 84, 85, 88, 90, 91 }

C grade { 48, 50, 52, 54 }

F normal fail { 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 74, 76, 77, 79, 80, 82, 83 }

F(-1) timedout fail { 75, 78 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 55, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92 }

C grade { }

F normal fail { }

F(-1) timedout fail { 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 5, 24, 44, 73 }

B grade { 1, 2, 3, 4, 6, 13, 20, 25, 26, 27, 31, 38, 45, 46, 84 }

C grade { }

F normal fail { 7, 8, 9, 14, 15, 16, 21, 22, 23, 32, 33, 34, 39, 40, 41, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

F(-1) timedout fail { 10, 11, 12, 17, 18, 19, 28, 29, 30, 35, 36, 37, 42, 43, 49, 50, 58, 59, 70, 71, 72, 80, 81, 82, 83, 85 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	20	37	23	473	31	25
N.S.	1	1.00	0.79	0.61	1.12	0.70	14.33	0.94	0.76
time (sec)	N/A	0.279	0.014	0.660	0.716	0.241	2.195	0.334	2.812

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	16	19	16	14	14	78	14	16
N.S.	1	0.84	1.00	0.84	0.74	0.74	4.11	0.74	0.84
time (sec)	N/A	0.249	0.004	0.119	0.221	0.238	1.250	0.324	2.272

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	18	16	21	14	153	22	15
N.S.	1	0.90	0.90	0.80	1.05	0.70	7.65	1.10	0.75
time (sec)	N/A	0.234	0.003	0.075	0.304	0.230	0.782	0.345	2.274

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	12	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.71	1.00	1.00
time (sec)	N/A	0.223	0.035	0.061	0.207	0.233	0.458	0.310	0.025

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	2	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.40	1.00	1.00
time (sec)	N/A	0.187	0.001	0.052	0.522	0.228	0.269	0.318	2.557

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	21	9	21	22	19	23	8
N.S.	1	1.00	2.62	1.12	2.62	2.75	2.38	2.88	1.00
time (sec)	N/A	0.220	0.018	0.086	0.316	0.231	0.109	0.317	2.748

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	20	51	21	37	48	0	38	26
N.S.	1	0.91	2.32	0.95	1.68	2.18	0.00	1.73	1.18
time (sec)	N/A	0.282	0.010	0.299	0.217	0.232	0.000	0.336	0.128

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	16	21	18	17	29	0	17	13
N.S.	1	0.84	1.11	0.95	0.89	1.53	0.00	0.89	0.68
time (sec)	N/A	0.254	0.005	0.095	0.223	0.219	0.000	0.344	2.490

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	75	33	51	72	0	47	39
N.S.	1	1.00	2.14	0.94	1.46	2.06	0.00	1.34	1.11
time (sec)	N/A	0.345	0.013	0.129	0.247	0.244	0.000	0.338	0.090

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	143	94	87	225	0	99	100
N.S.	1	1.00	1.83	1.21	1.12	2.88	0.00	1.27	1.28
time (sec)	N/A	0.305	0.294	1.267	0.299	0.273	0.000	0.339	2.410

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	116	57	53	152	0	59	65
N.S.	1	1.00	2.15	1.06	0.98	2.81	0.00	1.09	1.20
time (sec)	N/A	0.256	0.184	0.608	0.315	0.262	0.000	0.331	0.104

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	34	30	95	0	30	28
N.S.	1	1.00	2.50	0.94	0.83	2.64	0.00	0.83	0.78
time (sec)	N/A	0.223	0.172	0.252	0.326	0.269	0.000	0.328	0.093

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	73	66	17	18
N.S.	1	1.00	1.00	0.69	0.65	2.81	2.54	0.65	0.69
time (sec)	N/A	0.195	0.036	0.095	0.330	0.246	0.383	0.304	2.760

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	50	56	48	113	0	50	853
N.S.	1	1.00	1.19	1.33	1.14	2.69	0.00	1.19	20.31
time (sec)	N/A	0.220	0.058	0.210	0.334	0.261	0.000	0.335	2.906

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	77	140	95	105	274	0	103	1138
N.S.	1	1.24	2.26	1.53	1.69	4.42	0.00	1.66	18.35
time (sec)	N/A	0.262	0.430	0.409	0.339	0.276	0.000	0.329	3.022

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	125	204	155	200	592	0	178	833
N.S.	1	1.33	2.17	1.65	2.13	6.30	0.00	1.89	8.86
time (sec)	N/A	0.318	1.320	0.821	0.337	0.317	0.000	0.329	6.042

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	114	77	94	112	285	0	119	681
N.S.	1	1.30	0.88	1.07	1.27	3.24	0.00	1.35	7.74
time (sec)	N/A	0.332	0.177	1.402	0.333	0.273	0.000	0.320	2.788

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	74	52	61	62	211	0	80	126
N.S.	1	1.23	0.87	1.02	1.03	3.52	0.00	1.33	2.10
time (sec)	N/A	0.272	0.108	0.653	0.319	0.265	0.000	0.321	2.630

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	41	37	36	33	177	0	50	108
N.S.	1	1.02	0.92	0.90	0.82	4.42	0.00	1.25	2.70
time (sec)	N/A	0.238	0.087	0.206	0.311	0.275	0.000	0.340	3.109

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	10924	37	24
N.S.	1	1.00	0.97	0.70	0.67	5.43	364.13	1.23	0.80
time (sec)	N/A	0.186	0.476	0.127	0.322	0.263	18.672	0.319	2.624

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	39	38	228	0	55	34
N.S.	1	1.00	0.98	0.95	0.93	5.56	0.00	1.34	0.83
time (sec)	N/A	0.222	0.104	0.293	0.312	0.268	0.000	0.351	2.356

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	57	70	396	0	90	67
N.S.	1	1.00	0.97	0.93	1.15	6.49	0.00	1.48	1.10
time (sec)	N/A	0.278	0.185	0.510	0.318	0.281	0.000	0.352	2.382

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	83	127	610	0	156	101
N.S.	1	1.00	1.01	0.93	1.43	6.85	0.00	1.75	1.13
time (sec)	N/A	0.307	0.298	0.764	0.310	0.303	0.000	0.330	2.924

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	100	79	35	89	71	85	60	75
N.S.	1	1.02	0.81	0.36	0.91	0.72	0.87	0.61	0.77
time (sec)	N/A	0.327	0.149	0.182	0.328	0.284	0.803	0.373	0.450

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	8	14	6	4
N.S.	1	1.00	1.00	1.25	1.50	2.00	3.50	1.50	1.00
time (sec)	N/A	0.200	0.002	0.086	0.210	0.230	0.277	0.308	2.657

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	14	14	25	34	14	10
N.S.	1	1.00	1.31	1.08	1.08	1.92	2.62	1.08	0.77
time (sec)	N/A	0.215	0.003	0.116	0.243	0.246	0.804	0.318	2.334

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	20	20	37	54	20	17
N.S.	1	1.00	1.29	0.95	0.95	1.76	2.57	0.95	0.81
time (sec)	N/A	0.223	0.004	0.158	0.222	0.239	1.828	0.310	2.262

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	111	78	91	259	0	96	86
N.S.	1	1.00	1.42	1.00	1.17	3.32	0.00	1.23	1.10
time (sec)	N/A	0.280	0.330	2.002	0.293	0.271	0.000	0.299	0.146

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	86	50	67	191	0	65	51
N.S.	1	1.00	1.54	0.89	1.20	3.41	0.00	1.16	0.91
time (sec)	N/A	0.256	0.156	0.898	0.291	0.255	0.000	0.317	2.773

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	50	134	0	41	30
N.S.	1	1.00	1.00	0.87	1.32	3.53	0.00	1.08	0.79
time (sec)	N/A	0.216	0.028	0.434	0.304	0.255	0.000	0.321	0.124

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	21	39	95	55508	31	21
N.S.	1	1.00	1.00	0.72	1.34	3.28	1914.07	1.07	0.72
time (sec)	N/A	0.197	0.011	0.188	0.286	0.249	61.710	0.321	0.110

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	47	64	119	0	57	414
N.S.	1	1.00	0.93	1.15	1.56	2.90	0.00	1.39	10.10
time (sec)	N/A	0.219	0.049	0.392	0.300	0.254	0.000	0.321	2.687

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	73	152	82	92	186	0	85	483
N.S.	1	1.24	2.58	1.39	1.56	3.15	0.00	1.44	8.19
time (sec)	N/A	0.266	0.316	0.919	0.300	0.280	0.000	0.297	2.980

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	117	215	137	145	270	0	127	969
N.S.	1	1.30	2.39	1.52	1.61	3.00	0.00	1.41	10.77
time (sec)	N/A	0.331	0.846	1.763	0.308	0.286	0.000	0.329	2.682

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	116	76	92	97	273	0	104	1036
N.S.	1	1.33	0.87	1.06	1.11	3.14	0.00	1.20	11.91
time (sec)	N/A	0.376	0.175	1.248	0.312	0.286	0.000	0.315	3.122

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	74	52	59	54	213	0	72	291
N.S.	1	1.23	0.87	0.98	0.90	3.55	0.00	1.20	4.85
time (sec)	N/A	0.271	0.095	0.557	0.310	0.291	0.000	0.308	2.710

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	31	183	0	48	425
N.S.	1	1.00	0.95	0.89	0.82	4.82	0.00	1.26	11.18
time (sec)	N/A	0.274	0.434	0.183	0.328	0.267	0.000	0.333	2.905

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	10924	37	24
N.S.	1	1.00	0.97	0.70	0.67	5.43	364.13	1.23	0.80
time (sec)	N/A	0.186	0.016	0.000	0.296	0.266	20.820	0.332	0.004

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	33	32	216	0	36	30
N.S.	1	1.00	1.03	0.89	0.86	5.84	0.00	0.97	0.81
time (sec)	N/A	0.228	0.313	0.542	0.294	0.257	0.000	0.345	2.540

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	49	48	276	0	71	51
N.S.	1	1.00	0.98	0.88	0.86	4.93	0.00	1.27	0.91
time (sec)	N/A	0.278	0.347	1.187	0.327	0.265	0.000	0.341	2.768

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	80	78	74	348	0	104	84
N.S.	1	1.00	1.01	0.99	0.94	4.41	0.00	1.32	1.06
time (sec)	N/A	0.300	0.515	2.431	0.332	0.269	0.000	0.327	2.363

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	70	60	72	326	0	69	52
N.S.	1	1.00	1.08	0.92	1.11	5.02	0.00	1.06	0.80
time (sec)	N/A	0.289	5.108	0.602	0.304	0.284	0.000	0.309	2.401

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	120	106	117	186	616	0	149	123
N.S.	1	1.12	0.99	1.09	1.74	5.76	0.00	1.39	1.15
time (sec)	N/A	0.462	5.399	1.421	0.392	0.292	0.000	0.305	3.057

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	16	15	14	13	31	63	46	26
N.S.	1	0.47	0.44	0.41	0.38	0.91	1.85	1.35	0.76
time (sec)	N/A	0.174	0.060	0.141	0.397	0.253	0.352	0.332	2.797

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	35	35	27	26	57	218	59	40
N.S.	1	0.64	0.64	0.49	0.47	1.04	3.96	1.07	0.73
time (sec)	N/A	0.239	0.103	0.301	0.374	0.235	1.179	0.326	2.363

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	56	51	35	41	81	439	68	53
N.S.	1	0.79	0.72	0.49	0.58	1.14	6.18	0.96	0.75
time (sec)	N/A	0.340	0.174	0.573	0.305	0.238	4.223	0.325	2.389

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	10	4	0	24	10
N.S.	1	1.00	1.00	1.08	0.83	0.33	0.00	2.00	0.83
time (sec)	N/A	0.246	0.024	0.470	0.316	0.242	0.000	0.315	0.037

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	12	0	0	28	39
N.S.	1	1.00	1.00	1.00	0.86	0.00	0.00	2.00	2.79
time (sec)	N/A	0.241	0.016	0.402	0.314	0.000	0.000	0.329	2.690

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	19	11	11	0	45	0
N.S.	1	1.00	0.79	0.66	0.38	0.38	0.00	1.55	0.00
time (sec)	N/A	0.291	0.034	0.543	0.349	0.240	0.000	0.361	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	23	0	0	0	55	0
N.S.	1	1.00	0.76	0.70	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.299	0.038	0.520	0.000	0.000	0.000	0.347	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	28	14	35	19	0	21	0
N.S.	1	1.00	1.87	0.93	2.33	1.27	0.00	1.40	0.00
time (sec)	N/A	0.249	0.032	0.403	0.515	0.249	0.000	0.315	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	30	34	17	0	0	26	0
N.S.	1	1.00	1.76	2.00	1.00	0.00	0.00	1.53	0.00
time (sec)	N/A	0.253	0.036	0.459	0.524	0.000	0.000	0.328	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	51	37	300	44	0	78	0
N.S.	1	1.00	1.59	1.16	9.38	1.38	0.00	2.44	0.00
time (sec)	N/A	0.309	0.054	0.670	0.479	0.238	0.000	0.393	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	53	39	284	0	0	90	0
N.S.	1	1.00	1.47	1.08	7.89	0.00	0.00	2.50	0.00
time (sec)	N/A	0.307	0.042	0.635	0.331	0.000	0.000	0.377	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	0	0	0	7
N.S.	1	1.00	1.22	4.56	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.167	0.038	2.163	0.000	0.000	0.000	0.000	0.014

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	75	0	0	0	0	0
N.S.	1	1.00	1.06	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.036	2.314	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	46	49	0	0	0	0	0
N.S.	1	1.00	1.10	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.062	2.461	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	46	39	101	0	0	0	0	0
N.S.	1	1.07	0.91	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.046	1.094	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	92	66	110	0	0	0	0	0
N.S.	1	1.03	0.74	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	0.057	1.324	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	122	123	192	0	0	0	0	0
N.S.	1	1.01	1.02	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	0.360	2.966	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	87	0	0	0
N.S.	1	1.00	1.22	4.56	0.00	9.67	0.00	0.00	0.00
time (sec)	N/A	0.162	0.030	0.566	0.000	0.091	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	62	0	41	0	0	0
N.S.	1	1.00	1.03	1.94	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.226	0.035	0.578	0.000	0.075	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	46	48	0	276	0	0	0
N.S.	1	1.00	1.10	1.14	0.00	6.57	0.00	0.00	0.00
time (sec)	N/A	0.253	0.050	0.502	0.000	0.092	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	35	70	0	247	0	0	0
N.S.	1	1.00	1.09	2.19	0.00	7.72	0.00	0.00	0.00
time (sec)	N/A	0.233	0.052	0.841	0.000	0.100	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	101	0	168	0	0	0
N.S.	1	1.00	0.77	1.80	0.00	3.00	0.00	0.00	0.00
time (sec)	N/A	0.325	0.035	1.114	0.000	0.085	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	75	73	0	775	0	0	0
N.S.	1	1.00	0.96	0.94	0.00	9.94	0.00	0.00	0.00
time (sec)	N/A	0.351	0.145	1.367	0.000	0.141	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	33	8	49	0	23	0
N.S.	1	1.00	1.00	3.67	0.89	5.44	0.00	2.56	0.00
time (sec)	N/A	0.182	0.008	0.384	0.302	0.249	0.000	0.381	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	57	11	89	0	0	0
N.S.	1	1.00	1.00	3.80	0.73	5.93	0.00	0.00	0.00
time (sec)	N/A	0.188	0.022	0.651	0.290	0.252	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	53	8	39	0	29	0
N.S.	1	1.00	1.00	5.89	0.89	4.33	0.00	3.22	0.00
time (sec)	N/A	0.185	0.009	0.674	0.288	0.241	0.000	0.362	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	541	121	101	0	809	0	307	926
N.S.	1	1.11	0.25	0.21	0.00	1.66	0.00	0.63	1.90
time (sec)	N/A	1.263	2.847	0.741	0.000	0.353	0.000	0.396	3.564

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	156	109	72	0	817	0	299	938
N.S.	1	1.54	1.08	0.71	0.00	8.09	0.00	2.96	9.29
time (sec)	N/A	0.311	0.441	0.668	0.000	0.353	0.000	0.384	3.143

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	200	45	126	0	117	0	170	214
N.S.	1	0.68	0.15	0.43	0.00	0.40	0.00	0.58	0.73
time (sec)	N/A	0.468	5.025	0.501	0.000	0.265	0.000	0.531	3.207

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	25	24	21	20	43	78	53	20
N.S.	1	0.56	0.53	0.47	0.44	0.96	1.73	1.18	0.44
time (sec)	N/A	0.178	1.067	0.262	0.295	0.248	0.628	0.335	2.847

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	130	150	0	0	0	0	1520
N.S.	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	3.08
time (sec)	N/A	0.971	5.096	1.645	0.000	0.000	0.000	0.000	10.835

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	146	60	0	15483	0	0	184
N.S.	1	1.00	0.85	0.35	0.00	90.54	0.00	0.00	1.08
time (sec)	N/A	0.494	5.055	2.006	0.000	1.629	0.000	0.000	3.511

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	245	257	172	76	0	665467	0	0	216
N.S.	1	1.05	0.70	0.31	0.00	2716.19	0.00	0.00	0.88
time (sec)	N/A	0.665	5.091	4.374	0.000	6.129	0.000	0.000	4.605

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	130	148	0	0	0	0	1518
N.S.	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	3.07
time (sec)	N/A	0.776	5.146	1.622	0.000	0.000	0.000	0.000	8.982

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	146	62	0	16679	0	0	184
N.S.	1	1.00	0.83	0.35	0.00	95.31	0.00	0.00	1.05
time (sec)	N/A	0.493	5.161	1.892	0.000	1.657	0.000	0.000	3.831

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	172	78	0	643291	0	0	216
N.S.	1	1.00	0.81	0.37	0.00	3020.15	0.00	0.00	1.01
time (sec)	N/A	0.484	3.965	4.313	0.000	6.081	0.000	0.000	3.730

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	378	62	0	789	0	0	535
N.S.	1	1.00	1.70	0.28	0.00	3.54	0.00	0.00	2.40
time (sec)	N/A	0.701	5.068	0.351	0.000	0.396	0.000	0.000	3.282

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	79	73	72	138	0	185	99
N.S.	1	1.00	0.95	0.88	0.87	1.66	0.00	2.23	1.19
time (sec)	N/A	0.339	5.078	1.354	0.310	0.260	0.000	0.343	3.358

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	141	67	0	885	0	0	1025
N.S.	1	1.00	1.09	0.52	0.00	6.86	0.00	0.00	7.95
time (sec)	N/A	0.417	5.041	3.133	0.000	0.383	0.000	0.000	3.380

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	378	62	0	748	0	0	403
N.S.	1	1.00	1.84	0.30	0.00	3.65	0.00	0.00	1.97
time (sec)	N/A	0.626	5.046	0.331	0.000	0.370	0.000	0.000	2.692

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	117	51	0	239	728	199	95
N.S.	1	1.00	1.65	0.72	0.00	3.37	10.25	2.80	1.34
time (sec)	N/A	0.479	1.359	0.414	0.000	0.289	8.914	0.362	3.158

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	69	64	176	0	166	0	222	241
N.S.	1	0.78	0.72	1.98	0.00	1.87	0.00	2.49	2.71
time (sec)	N/A	0.428	5.081	2.677	0.000	0.291	0.000	0.542	2.911

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	19	17	16	19	19	0	16	9
N.S.	1	1.12	1.00	0.94	1.12	1.12	0.00	0.94	0.53
time (sec)	N/A	0.200	0.022	0.613	0.209	0.246	0.000	0.301	2.438

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	45	40	43	95	90	0	38	0
N.S.	1	1.12	1.00	1.08	2.38	2.25	0.00	0.95	0.00
time (sec)	N/A	0.234	0.032	0.580	0.433	0.354	0.000	0.306	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	26	17	17	47	21	0	45	0
N.S.	1	1.30	0.85	0.85	2.35	1.05	0.00	2.25	0.00
time (sec)	N/A	0.262	0.059	0.576	0.383	0.248	0.000	0.299	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	81	67	0	24	0
N.S.	1	1.00	1.00	1.20	3.24	2.68	0.00	0.96	0.00
time (sec)	N/A	0.237	0.029	0.583	0.434	0.287	0.000	0.314	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	60	16	0	27	0
N.S.	1	1.00	1.00	0.91	5.45	1.45	0.00	2.45	0.00
time (sec)	N/A	0.207	0.026	0.479	0.293	0.255	0.000	0.302	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	14	8	39	17	0	33	0
N.S.	1	1.00	1.56	0.89	4.33	1.89	0.00	3.67	0.00
time (sec)	N/A	0.253	0.030	0.377	0.305	0.235	0.000	0.288	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	217	109	151	823	0	143	1281
N.S.	1	1.00	1.42	0.71	0.99	5.38	0.00	0.93	8.37
time (sec)	N/A	0.424	0.364	5.555	0.303	0.942	0.000	0.319	5.950

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	52	123	0	38	0
N.S.	1	1.00	1.00	0.00	1.16	2.73	0.00	0.84	0.00
time (sec)	N/A	0.255	0.023	0.000	0.604	1.575	0.000	0.337	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	39	0	0	24	0
N.S.	1	1.00	1.00	0.00	1.39	0.00	0.00	0.86	0.00
time (sec)	N/A	0.242	0.015	0.000	0.552	0.000	0.000	0.315	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	44	43	90	0	38	0
N.S.	1	1.00	1.00	0.98	0.96	2.00	0.00	0.84	0.00
time (sec)	N/A	0.244	0.029	0.781	0.225	0.319	0.000	0.297	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	31	0	67	0	24	0
N.S.	1	1.00	1.00	1.11	0.00	2.39	0.00	0.86	0.00
time (sec)	N/A	0.243	0.013	0.400	0.000	0.320	0.000	0.318	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	46	46	39	58	97	0	46	0
N.S.	1	0.98	0.98	0.83	1.23	2.06	0.00	0.98	0.00
time (sec)	N/A	0.251	0.029	0.701	0.385	0.247	0.000	0.370	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	42	74	0	27	0
N.S.	1	1.00	1.00	0.83	1.45	2.55	0.00	0.93	0.00
time (sec)	N/A	0.249	0.014	0.190	0.428	0.243	0.000	0.322	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [46] had the largest ratio of [1.12500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	1.00	16	0.438
2	A	8	7	0.84	16	0.438
3	A	5	5	0.90	16	0.312
4	A	6	6	1.00	16	0.375
5	A	3	3	1.00	16	0.188
6	A	6	6	1.00	14	0.429
7	A	8	8	0.91	14	0.571
8	A	6	5	0.84	16	0.312
9	A	10	10	1.00	16	0.625
10	A	6	5	1.00	15	0.333
11	A	6	5	1.00	15	0.333
12	A	6	5	1.00	15	0.333
13	A	5	4	1.00	13	0.308
14	A	7	6	1.00	13	0.462
15	A	8	7	1.24	15	0.467
16	A	9	8	1.33	15	0.533
17	A	8	7	1.30	15	0.467
18	A	7	6	1.23	15	0.400
19	A	6	5	1.02	15	0.333
20	A	4	3	1.00	10	0.300
21	A	5	4	1.00	15	0.267
22	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	4	1.00	15	0.267
24	A	11	10	1.02	13	0.769
25	A	6	5	1.00	10	0.500
26	A	6	5	1.00	10	0.500
27	A	6	5	1.00	10	0.500
28	A	5	4	1.00	15	0.267
29	A	5	4	1.00	15	0.267
30	A	5	4	1.00	15	0.267
31	A	4	3	1.00	13	0.231
32	A	6	5	1.00	13	0.385
33	A	7	6	1.24	15	0.400
34	A	8	7	1.30	15	0.467
35	A	8	7	1.33	15	0.467
36	A	7	6	1.23	15	0.400
37	A	6	5	1.00	15	0.333
38	A	4	3	1.00	10	0.300
39	A	5	4	1.00	15	0.267
40	A	5	4	1.00	15	0.267
41	A	5	4	1.00	15	0.267
42	A	8	7	1.00	10	0.700
43	A	10	9	1.12	10	0.900
44	A	4	3	0.47	8	0.375
45	A	7	6	0.64	8	0.750
46	A	10	9	0.79	8	1.125
47	A	6	6	1.00	12	0.500
48	A	6	6	1.00	10	0.600
49	A	8	8	1.00	12	0.667
50	A	8	8	1.00	10	0.800
51	A	6	6	1.00	12	0.500
52	A	6	6	1.00	10	0.600
53	A	8	8	1.00	12	0.667
54	A	8	8	1.00	10	0.800
55	A	2	2	1.00	10	0.200
56	A	4	4	1.00	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	4	1.00	12	0.333
58	A	8	8	1.07	10	0.800
59	A	12	12	1.03	12	1.000
60	A	11	11	1.01	12	0.917
61	A	2	2	1.00	10	0.200
62	A	4	4	1.00	12	0.333
63	A	4	4	1.00	12	0.333
64	A	5	5	1.00	10	0.500
65	A	7	7	1.00	12	0.583
66	A	7	7	1.00	12	0.583
67	A	4	3	1.00	13	0.231
68	A	4	3	1.00	21	0.143
69	A	4	3	1.00	15	0.200
70	A	10	9	1.11	10	0.900
71	A	5	4	1.54	11	0.364
72	A	9	8	0.68	8	1.000
73	A	5	4	0.56	10	0.400
74	A	3	3	1.00	10	0.300
75	A	6	5	1.00	10	0.500
76	A	6	5	1.05	10	0.500
77	A	3	3	1.00	11	0.273
78	A	6	5	1.00	11	0.455
79	A	6	5	1.00	11	0.455
80	A	3	3	1.00	8	0.375
81	A	6	5	1.00	8	0.625
82	A	6	5	1.00	8	0.625
83	A	3	3	1.00	10	0.300
84	A	10	9	1.00	10	0.900
85	A	10	9	0.78	10	0.900
86	A	7	6	1.12	11	0.545
87	A	7	6	1.12	15	0.400
88	A	10	9	1.30	15	0.600
89	A	6	5	1.00	15	0.333
90	A	6	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	9	8	1.00	15	0.533
92	A	6	5	1.00	15	0.333
93	A	8	7	1.00	15	0.467
94	A	7	6	1.00	15	0.400
95	A	8	7	1.00	15	0.467
96	A	7	6	1.00	15	0.400
97	A	8	7	0.98	15	0.467
98	A	7	6	1.00	15	0.400

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sin^6(x)}{a-a\cos^2(x)}\,dx$

3.1.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{\sin^6(x)}{a-a\cos^2(x)}\,dx=\frac{3x}{8a}-\frac{3\cos(x)\sin(x)}{8a}-\frac{\cos(x)\sin^3(x)}{4a}$$

output
```
3/8*x/a-3/8*cos(x)*sin(x)/a-1/4*cos(x)*sin(x)^3/a
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{\sin^6(x)}{a-a\cos^2(x)}\,dx=\frac{\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a}$$

input
```
Integrate[Sin[x]^6/(a - a*Cos[x]^2),x]
```

output
```
((3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32)/a
```

3.1.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(x)}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^6}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sin^4(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(x)^4dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)}{a}$$

input
```
Int[Sin[x]^6/(a - a*Cos[x]^2),x]
```

output
```
(-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4)/a
```

3.1.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.1.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{12x+\sin(4x)-8\sin(2x)}{32a}$
risch	$\frac{3x}{8a}+\frac{\sin(4x)}{32a}-\frac{\sin(2x)}{4a}$
default	$\frac{\frac{-\frac{5\left(\tan^{3}(x)\right)}{8}-\frac{3\tan(x)}{8}}{\left(\tan^{2}(x)+1\right)^{2}}+\frac{3\arctan(\tan(x))}{8}}{a}$
norman	$\frac{-\frac{3\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{4a}-\frac{17\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4a}-\frac{7\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{2a}+\frac{7\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{2a}+\frac{17\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{4a}+\frac{3\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{4a}+\frac{3x\tan\left(\frac{x}{2}\right)}{8a}+\frac{9x\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4a}+\frac{45x\left(\tan^{5}\right.}{8a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{6}\tan\left(\frac{x}{2}\right)}$

input
```
int(sin(x)^6/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*x+sin(4*x)-8*sin(2*x))/a
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{\sin^6(x)}{a - a\cos^2(x)}\,dx = \frac{\left(2\,\cos\left(x\right)^3 - 5\,\cos\left(x\right)\right)\sin\left(x\right) + 3\,x}{8\,a}$$

input
```
integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
1/8*((2*cos(x)^3 - 5*cos(x))*sin(x) + 3*x)/a
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(29) = 58.

Time = 2.19 (sec) , antiderivative size = 473, normalized size of antiderivative = 14.33

$$\begin{aligned}\int \frac{\sin^6(x)}{a - a\cos^2(x)}\,dx = {} & \frac{3x\tan^8\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & + \frac{12x\tan^6\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & + \frac{18x\tan^4\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & + \frac{12x\tan^2\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & + \frac{3x}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & + \frac{6\tan^7\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & + \frac{22\tan^5\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & - \frac{22\tan^3\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\ & - \frac{6\tan\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a}\end{aligned}$$

input
```
integrate(sin(x)**6/(a-a*cos(x)**2),x)
```

output
```
3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 3
2*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)
**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*t
an(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
 + 12*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
 + 32*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*
a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/2)**7/(8*a*tan(x/2)**8 +
 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 22*tan(x/
2)**5/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/
2)**2 + 8*a) - 22*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*t
an(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) - 6*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*
tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\sin^6(x)}{a - a\cos^2(x)}\,dx = -\frac{5\,\tan(x)^3 + 3\,\tan(x)}{8\left(a\tan(x)^4 + 2\,a\tan(x)^2 + a\right)} + \frac{3\,x}{8\,a}$$

input
```
integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/8*(5*tan(x)^3 + 3*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\sin^6(x)}{a - a\cos^2(x)}\,dx = \frac{3\,x}{8\,a} - \frac{5\,\tan(x)^3 + 3\,\tan(x)}{8\left(\tan(x)^2 + 1\right)^2 a}$$

input
```
integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
3/8*x/a - 1/8*(5*tan(x)^3 + 3*tan(x))/((tan(x)^2 + 1)^2*a)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{\sin^6(x)}{a - a\cos^2(x)}\,dx = \frac{\sin(4\,x)}{32\,a} - \frac{\sin(2\,x)}{4\,a} + \frac{3\,x}{8\,a}$$

input
```
int(sin(x)^6/(a - a*cos(x)^2),x)
```

output
```
sin(4*x)/(32*a) - sin(2*x)/(4*a) + (3*x)/(8*a)
```

3.2 $\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx$

3.2.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx = -\frac{\cos(x)}{a}+\frac{\cos^3(x)}{3a}$$

output
```
-cos(x)/a+1/3*cos(x)^3/a
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx = \frac{-\frac{3\cos(x)}{4}+\frac{1}{12}\cos(3x)}{a}$$

input
```
Integrate[Sin[x]^5/(a - a*Cos[x]^2),x]
```

output
```
((-3*Cos[x])/4 + Cos[3*x]/12)/a
```

3.2.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 25, 3654, 25, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(x)}{a-a\cos^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)^5}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)^5}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$-\frac{\int -\sin^3(x)dx}{a}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \sin^3(x)dx}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sin(x)^3dx}{a}$$
$$\Big\downarrow \text{ 3113}$$
$$-\frac{\int \left(1-\cos^2(x)\right)d\cos(x)}{a}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{\cos(x)-\frac{\cos^3(x)}{3}}{a}$$

input
```
Int[Sin[x]^5/(a - a*Cos[x]^2),x]
```

output
```
-((Cos[x] - Cos[x]^3/3)/a)
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.2.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^3(x)\right)}{3}-\cos(x)}{a}$	16
default	$\frac{\frac{\left(\cos^3(x)\right)}{3}-\cos(x)}{a}$	16
parallelrisch	$\frac{-8-9\cos(x)+\cos(3x)}{12a}$	16
risch	$-\frac{3\cos(x)}{4a}+\frac{\cos(3x)}{12a}$	18
norman	$\frac{-\frac{4\tan\left(\frac{x}{2}\right)}{3a}-\frac{20\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{4\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}-\frac{28\left(\tan^5\left(\frac{x}{2}\right)\right)}{3a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5\tan\left(\frac{x}{2}\right)}$	61

input
```
int(sin(x)^5/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/3*cos(x)^3-cos(x))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^5(x)}{a - a\cos^2(x)}\,dx = \frac{\cos(x)^3 - 3\cos(x)}{3\,a}$$

input
```
integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
1/3*(cos(x)^3 - 3*cos(x))/a
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(12) = 24.

Time = 1.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.11

$$\int \frac{\sin^5(x)}{a - a\cos^2(x)}\,dx = -\frac{12\tan^2\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a} - \frac{4}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a}$$

input
```
integrate(sin(x)**5/(a-a*cos(x)**2),x)
```

output
```
-12*tan(x/2)**2/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a
) - 4/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^5(x)}{a - a\cos^2(x)}\,dx = \frac{\cos(x)^3 - 3\cos(x)}{3a}$$

input
```
integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
1/3*(cos(x)^3 - 3*cos(x))/a
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^5(x)}{a - a\cos^2(x)}\,dx = \frac{\cos(x)^3 - 3\cos(x)}{3a}$$

input
```
integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
1/3*(cos(x)^3 - 3*cos(x))/a
```

3.2.9 Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sin^5(x)}{a - a\cos^2(x)}\,dx = -\frac{3\cos(x) - \cos(x)^3}{3a}$$

input
```
int(sin(x)^5/(a - a*cos(x)^2),x)
```

output
```
-(3*cos(x) - cos(x)^3)/(3*a)
```

3.3 $\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx$

3.3.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx=\frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}$$

output
```
1/2*x/a-1/2*cos(x)*sin(x)/a
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx=\frac{\frac{x}{2}-\frac{1}{4}\sin(2x)}{a}$$

input
```
Integrate[Sin[x]^4/(a - a*Cos[x]^2),x]
```

output
```
(x/2 - Sin[2*x]/4)/a
```

3.3.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^4}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sin^2(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin(x)^2dx}{a}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)}{a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)}{a}$$

input
```
Int[Sin[x]^4/(a - a*Cos[x]^2),x]
```

output
```
(x/2 - (Cos[x]*Sin[x])/2)/a
```

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.3.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{2x-\sin(2x)}{4a}$	16
risch	$\frac{x}{2a}-\frac{\sin(2x)}{4a}$	17
default	$\frac{-\frac{\tan(x)}{2\left(\tan^2(x)+1\right)}+\frac{\arctan(\tan(x))}{2}}{a}$	23
norman	$\frac{\frac{\tan^6\left(\frac{x}{2}\right)}{a}+\frac{\tan^8\left(\frac{x}{2}\right)}{a}-\frac{\tan^2\left(\frac{x}{2}\right)}{a}-\frac{\tan^4\left(\frac{x}{2}\right)}{a}+\frac{x\tan\left(\frac{x}{2}\right)}{2a}+\frac{2x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{2x\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^9\left(\frac{x}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4\tan\left(\frac{x}{2}\right)}$	119

input
```
int(sin(x)^4/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x-sin(2*x))/a
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sin^4(x)}{a - a\cos^2(x)}\, dx = -\frac{\cos(x)\sin(x) - x}{2\,a}$$

input
```
integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(cos(x)*sin(x) - x)/a
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(14) = 28.

Time = 0.78 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\int \frac{\sin^4(x)}{a - a\cos^2(x)}\, dx = \frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2\tan^3\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} - \frac{2\tan\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a}$$

input
```
integrate(sin(x)**4/(a-a*cos(x)**2),x)
```

output
```
x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/
(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x
/2)**2 + 2*a) + 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) -
2*tan(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\sin^4(x)}{a - a\cos^2(x)}\,dx = \frac{x}{2\,a} - \frac{\tan(x)}{2\,\left(a\tan(x)^2 + a\right)}$$

input
```
integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*x/a - 1/2*tan(x)/(a*tan(x)^2 + a)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin^4(x)}{a - a\cos^2(x)}\,dx = \frac{x}{2\,a} - \frac{\tan(x)}{2\,\left(\tan(x)^2 + 1\right)a}$$

input
```
integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
1/2*x/a - 1/2*tan(x)/((tan(x)^2 + 1)*a)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\sin^4(x)}{a - a\cos^2(x)}\,dx = \frac{2\,x - \sin(2\,x)}{4\,a}$$

input
```
int(sin(x)^4/(a - a*cos(x)^2),x)
```

output
```
(2*x - sin(2*x))/(4*a)
```

3.4 $\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx$

3.4.1 Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx = -\frac{\cos(x)}{a}$$

output
```
-cos(x)/a
```

3.4.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx = -\frac{\cos(x)}{a}$$

input
```
Integrate[Sin[x]^3/(a - a*Cos[x]^2),x]
```

output
```
-(Cos[x]/a)
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 25, 3654, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)^3}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)^3}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$-\frac{\int -\sin(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sin(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(x)dx}{a}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\cos(x)}{a}$$

input
```
Int[Sin[x]^3/(a - a*Cos[x]^2),x]
```

output
```
-(Cos[x]/a)
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.4.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\frac{\cos(x)}{a}$	8
default	$-\frac{\cos(x)}{a}$	8
risch	$-\frac{\cos(x)}{a}$	8
parallelrisch	$\frac{-\cos(x)-1}{a}$	11
norman	$\frac{\frac{2\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}+\frac{4\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}\tan\left(\frac{x}{2}\right)}$	52

input
```
int(sin(x)^3/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)/a
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a - a\cos^2(x)}\,dx = -\frac{\cos(x)}{a}$$

input
```
integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
-cos(x)/a
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{\sin^3(x)}{a - a\cos^2(x)}\,dx = -\frac{2}{a\tan^2\left(\frac{x}{2}\right) + a}$$

input
```
integrate(sin(x)**3/(a-a*cos(x)**2),x)
```

output
```
-2/(a*tan(x/2)**2 + a)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a - a\cos^2(x)}\,dx = -\frac{\cos(x)}{a}$$

input
```
integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
-cos(x)/a
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a - a\cos^2(x)}\, dx = -\frac{\cos(x)}{a}$$

input
```
integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
-cos(x)/a
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a - a\cos^2(x)}\, dx = -\frac{\cos(x)}{a}$$

input
```
int(sin(x)^3/(a - a*cos(x)^2),x)
```

output
```
-cos(x)/a
```

3.5 $\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx$

3.5.1 Optimal result

Integrand size = 16, antiderivative size = 5

$$\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx=\frac{x}{a}$$

output
```
x/a
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx=\frac{x}{a}$$

input
```
Integrate[Sin[x]^2/(a - a*Cos[x]^2),x]
```

output
```
x/a
```

3.5.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3654, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a-a\cos^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{\int 1dx}{a}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{x}{a}$$

input
```
Int[Sin[x]^2/(a - a*Cos[x]^2),x]
```

output
```
x/a
```

3.5.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.5.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{x}{a}$	6
default	$\frac{\arctan(\tan(x))}{a}$	8
norman	$\frac{\frac{x\tan\left(\frac{x}{2}\right)}{a}+\frac{x\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{2x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)}$	51

input
```
int(sin(x)^2/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx=\frac{x}{a}$$

input
```
integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
x/a
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.40

$$\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx=\frac{x}{a}$$

input
```
integrate(sin(x)**2/(a-a*cos(x)**2),x)
```

output
```
x/a
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{a - a\cos^2(x)}\,dx = \frac{x}{a}$$

input
```
integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
x/a
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{a - a\cos^2(x)}\,dx = \frac{x}{a}$$

input
```
integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
x/a
```

3.5.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{a - a\cos^2(x)}\,dx = \frac{x}{a}$$

input
```
int(sin(x)^2/(a - a*cos(x)^2),x)
```

output
```
x/a
```

3.6 $\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 8

$$\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{a}$$

output
```
-arctanh(cos(x))/a
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx = \frac{-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sin[x]/(a - a*Cos[x]^2),x]
```

output
```
(-Log[Cos[x/2]] + Log[Sin[x/2]])/a
```

3.6.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 25, 3654, 25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$-\frac{\int -\csc(x)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \csc(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc(x)dx}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\text{arctanh}(\cos(x))}{a}$$

input
```
Int[Sin[x]/(a - a*Cos[x]^2),x]
```

output
```
-(ArcTanh[Cos[x]]/a)
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.6.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}(\cos(x))}{a}$	9
default	$-\frac{\operatorname{arctanh}(\cos(x))}{a}$	9
norman	$\frac{\ln(\tan(\frac{x}{2}))}{a}$	10
parallelrisch	$\frac{\ln(\tan(\frac{x}{2}))}{a}$	10
risch	$\frac{\ln(e^{ix}-1)}{a}-\frac{\ln(e^{ix}+1)}{a}$	27

input
```
int(sin(x)/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(cos(x))/a
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \frac{\sin(x)}{a - a\cos^2(x)}\,dx = -\frac{\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)}{2\,a}$$

input
```
integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(log(1/2*cos(x) + 1/2) - log(-1/2*cos(x) + 1/2))/a
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{\sin(x)}{a - a\cos^2(x)}\,dx = \frac{\log(\cos(x) - 1)}{2a} - \frac{\log(\cos(x) + 1)}{2a}$$

input
```
integrate(sin(x)/(a-a*cos(x)**2),x)
```

output
```
log(cos(x) - 1)/(2*a) - log(cos(x) + 1)/(2*a)
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{\sin(x)}{a - a\cos^2(x)}\,dx = -\frac{\log(\cos(x) + 1)}{2\,a} + \frac{\log(\cos(x) - 1)}{2\,a}$$

input
```
integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log(cos(x) + 1)/a + 1/2*log(cos(x) - 1)/a
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(8) = 16$.

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{\sin(x)}{a - a\cos^2(x)}\,dx = -\frac{\log(\cos(x) + 1)}{2a} + \frac{\log(-\cos(x) + 1)}{2a}$$

input
```
integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
-1/2*log(cos(x) + 1)/a + 1/2*log(-cos(x) + 1)/a
```

3.6.9 Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a - a\cos^2(x)}\,dx = -\frac{\operatorname{atanh}(\cos(x))}{a}$$

input
```
int(sin(x)/(a - a*cos(x)^2),x)
```

output
```
-atanh(cos(x))/a
```

3.7 $\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx$

3.7.1 Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx=-\frac{\operatorname{arctanh}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}$$

output
```
-1/2*arctanh(cos(x))/a-1/2*cot(x)*csc(x)/a
```

3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx=\frac{-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Csc[x]/(a - a*Cos[x]^2),x]
```

output
```
(-1/8*Csc[x/2]^2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8)/a
```

3.7.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 25, 3654, 25, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(x+\frac{\pi}{2}\right)\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$-\frac{\int -\csc^3(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \csc^3(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(x)^3dx}{a}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)}{a}$$

input
```
Int[Csc[x]/(a - a*Cos[x]^2),x]
```

output
```
(-1/2*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/2)/a
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$\frac{-\csc(x)\cot(x)+\ln(-\cot(x)+\csc(x))}{2a}$	21
default	$\frac{\frac{1}{4+4\cos(x)}-\frac{\ln(1+\cos(x))}{4}+\frac{1}{4\cos(x)-4}+\frac{\ln(\cos(x)-1)}{4}}{a}$	36
norman	$\frac{-\frac{1}{8a}+\frac{\tan^4\left(\frac{x}{2}\right)}{8a}}{\tan\left(\frac{x}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	36
risch	$\frac{e^{3ix}+e^{ix}}{\left(e^{2ix}-1\right)^2 a}-\frac{\ln\left(e^{ix}+1\right)}{2a}+\frac{\ln\left(e^{ix}-1\right)}{2a}$	52

input
```
int(csc(x)/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-csc(x)*cot(x)+ln(-cot(x)+csc(x)))/a
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{\csc(x)}{a - a\cos^2(x)}\,dx = -\frac{(\cos(x)^2 - 1)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - (\cos(x)^2 - 1)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 2\cos(x)}{4\,(a\cos(x)^2 - a)}$$

input
```
integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x
) + 1/2) - 2*cos(x))/(a*cos(x)^2 - a)
```

3.7.6 Sympy [F]

$$\int \frac{\csc(x)}{a - a\cos^2(x)}\,dx = -\frac{\int \frac{\csc(x)}{\cos^2(x)-1}\,dx}{a}$$

input
```
integrate(csc(x)/(a-a*cos(x)**2),x)
```

output
```
-Integral(csc(x)/(cos(x)**2 - 1), x)/a
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\csc(x)}{a - a\cos^2(x)}\,dx = \frac{\cos(x)}{2\left(a\cos(x)^2 - a\right)} - \frac{\log(\cos(x) + 1)}{4\,a} + \frac{\log(\cos(x) - 1)}{4\,a}$$

input
```
integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*cos(x)/(a*cos(x)^2 - a) - 1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/
a
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\csc(x)}{a - a\cos^2(x)}\,dx = -\frac{\log(\cos(x) + 1)}{4\,a} + \frac{\log(-\cos(x) + 1)}{4\,a} + \frac{\cos(x)}{2\left(\cos(x)^2 - 1\right)a}$$

input
```
integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
-1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2*cos(x)/((cos(x)^2 -
1)*a)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(x)}{a - a\cos^2(x)}\,dx = -\frac{\cos(x)}{2\left(a - a\cos(x)^2\right)} - \frac{\operatorname{atanh}(\cos(x))}{2\,a}$$

input
```
int(1/(sin(x)*(a - a*cos(x)^2)),x)
```

output
```
- cos(x)/(2*(a - a*cos(x)^2)) - atanh(cos(x))/(2*a)
```

3.8 $\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx$

3.8.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx=-\frac{\cot(x)}{a}-\frac{\cot^3(x)}{3a}$$

output
```
-cot(x)/a-1/3*cot(x)^3/a
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx=\frac{-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)}{a}$$

input
```
Integrate[Csc[x]^2/(a - a*Cos[x]^2),x]
```

output
```
((-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3)/a
```

3.8.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^2\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \; 3654$$

$$\frac{\int \csc^4(x)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc(x)^4dx}{a}$$

$$\Big\downarrow \; 4254$$

$$-\frac{\int \left(\cot^2(x)+1\right)d\cot(x)}{a}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\cot^3(x)}{3}+\cot(x)}{a}$$

input
```
Int[Csc[x]^2/(a - a*Cos[x]^2),x]
```

output
```
-((Cot[x] + Cot[x]^3/3)/a)
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{1}{\tan(x)}-\frac{1}{3\tan(x)^{3}}}{a}$	18
parallelrisch	$-\frac{\cot(x)\left(3\left(\csc^{2}(x)\right)-2\left(\cot^{2}(x)\right)\right)}{3a}$	21
risch	$\frac{4i\left(3\,\mathrm{e}^{2ix}-1\right)}{3\left(\mathrm{e}^{2ix}-1\right)^{3}a}$	25
norman	$\frac{-\frac{1}{24a}-\frac{3\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8a}+\frac{3\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^{6}\left(\frac{x}{2}\right)}{24a}}{\tan\left(\frac{x}{2}\right)^{3}}$	47

input
```
int(csc(x)^2/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/tan(x)-1/3/tan(x)^3)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx = -\frac{2\cos(x)^3-3\cos(x)}{3\left(a\cos(x)^2-a\right)\sin(x)}$$

input
```
integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/3*(2*cos(x)^3 - 3*cos(x))/((a*cos(x)^2 - a)*sin(x))
```

3.8.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx = -\frac{\int \frac{\csc^2(x)}{\cos^2(x)-1}\,dx}{a}$$

input
```
integrate(csc(x)**2/(a-a*cos(x)**2),x)
```

output
```
-Integral(csc(x)**2/(cos(x)**2 - 1), x)/a
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx = -\frac{3\tan(x)^2+1}{3a\tan(x)^3}$$

input
```
integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*tan(x)^2 + 1)/(a*tan(x)^3)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx = -\frac{3\,\tan(x)^2+1}{3\,a\tan(x)^3}$$

input
```
integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
-1/3*(3*tan(x)^2 + 1)/(a*tan(x)^3)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx = -\frac{\cot(x)\,\left(\cot(x)^2+3\right)}{3\,a}$$

input
```
int(1/(sin(x)^2*(a - a*cos(x)^2)),x)
```

output
```
-(cot(x)*(cot(x)^2 + 3))/(3*a)
```

3.9 $\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx$

3.9.1 Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx = -\frac{3\text{arctanh}(\cos(x))}{8a} - \frac{3\cot(x)\csc(x)}{8a} - \frac{\cot(x)\csc^3(x)}{4a}$$

output
```
-3/8*arctanh(cos(x))/a-3/8*cot(x)*csc(x)/a-1/4*cot(x)*csc(x)^3/a
```

3.9.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx = \frac{-\frac{3}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{3}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Csc[x]^3/(a - a*Cos[x]^2),x]
```

output
```
((-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2
]])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64)/a
```

3.9.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 25, 3654, 25, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\cos\left(x+\frac{\pi}{2}\right)^3\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^3\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{ 3654}$$

$$-\frac{\int -\csc^5(x)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \csc^5(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc(x)^5dx}{a}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)}{a}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3}{4}\left(-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)}{a}$$

input
```
Int[Csc[x]^3/(a - a*Cos[x]^2),x]
```

output
```
(-1/4*(Cot[x]*Csc[x]^3) + (3*(-1/2*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/2))/4
)/a
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.9.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
parallelrisch	$\frac{3\ln(-\cot(x)+\csc(x))+3\csc(x)\left(\cot^3(x)\right)-5\cot(x)\left(\csc^3(x)\right)}{8a}$	33
default	$\frac{\frac{1}{16(1+\cos(x))^2}+\frac{3}{16(1+\cos(x))}-\frac{3\ln(1+\cos(x))}{16}-\frac{1}{16(\cos(x)-1)^2}+\frac{3}{16(\cos(x)-1)}+\frac{3\ln(\cos(x)-1)}{16}}{a}$	52
norman	$\frac{-\frac{1}{64a}-\frac{\tan^2\left(\frac{x}{2}\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{8a}+\frac{\tan^8\left(\frac{x}{2}\right)}{64a}}{\tan\left(\frac{x}{2}\right)^4}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$	58
risch	$\frac{3\,\mathrm{e}^{7ix}-11\,\mathrm{e}^{5ix}-11\,\mathrm{e}^{3ix}+3\,\mathrm{e}^{ix}}{4(\mathrm{e}^{2ix}-1)^4a}+\frac{3\ln(\mathrm{e}^{ix}-1)}{8a}-\frac{3\ln(\mathrm{e}^{ix}+1)}{8a}$	71

input
```
int(csc(x)^3/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(3*ln(-cot(x)+csc(x))+3*csc(x)*cot(x)^3-5*cot(x)*csc(x)^3)
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.06

$$\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx$$
$$=\frac{6\cos(x)^3-3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(-\frac{1}{2}\cos(\right.}{16\left(a\cos(x)^4-2a\cos(x)^2+a\right)}$$

input
```
integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3
*(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(a*cos(x)
^4 - 2*a*cos(x)^2 + a)
```

3.9.6 Sympy [F]

$$\int \frac{\csc^3(x)}{a - a\cos^2(x)}\,dx = -\frac{\int \frac{\csc^3(x)}{\cos^2(x)-1}\,dx}{a}$$

input
```
integrate(csc(x)**3/(a-a*cos(x)**2),x)
```

output
```
-Integral(csc(x)**3/(cos(x)**2 - 1), x)/a
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{\csc^3(x)}{a - a\cos^2(x)}\,dx = \frac{3\cos(x)^3 - 5\cos(x)}{8\left(a\cos(x)^4 - 2\,a\cos(x)^2 + a\right)} - \frac{3\log(\cos(x)+1)}{16\,a} + \frac{3\log(\cos(x)-1)}{16\,a}$$

input
```
integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
1/8*(3*cos(x)^3 - 5*cos(x))/(a*cos(x)^4 - 2*a*cos(x)^2 + a) - 3/16*log(cos
(x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\csc^3(x)}{a - a\cos^2(x)}\,dx = -\frac{3\log(\cos(x)+1)}{16\,a} + \frac{3\log(-\cos(x)+1)}{16\,a} + \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^2 - 1\right)^2 a}$$

input
```
integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
-3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^3 - 5*co
s(x))/((cos(x)^2 - 1)^2*a)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx = -\frac{3\,\mathrm{atanh}(\cos{(x)})}{8\,a}-\frac{\frac{5\,\cos(x)}{8}-\frac{3\cos(x)^3}{8}}{a\cos{(x)}^4-2\,a\cos{(x)}^2+a}$$

input
```
int(1/(sin(x)^3*(a - a*cos(x)^2)),x)
```

output
```
- (3*atanh(cos(x)))/(8*a) - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(a - 2*a*cos(x
)^2 + a*cos(x)^4)
```

3.10 $\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx$

3.10.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx=-\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{(a^2+3ab+3b^2)\cos(x)}{b^3}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}$$

output
```
(a^2+3*a*b+3*b^2)*cos(x)/b^3-1/3*(a+3*b)*cos(x)^3/b^2+1/5*cos(x)^5/b-(a+b)
^3*arctan(cos(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.83

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx=-\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{(8a^2+22ab+19b^2)\cos(x)}{8b^3}-\frac{(4a+9b)\cos(3x)}{48b^2}+\frac{\cos(5x)}{80b}$$

input
```
Integrate[Sin[x]^7/(a + b*Cos[x]^2),x]
```

output
```
-(((a + b)^3*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/(Sqrt[a]*b^
(7/2)) - ((a + b)^3*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/(Sq
rt[a]*b^(7/2)) + ((8*a^2 + 22*a*b + 19*b^2)*Cos[x])/(8*b^3) - ((4*a + 9*b)
*Cos[3*x])/(48*b^2) + Cos[5*x]/(80*b)
```

3.10.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)^7}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)^7}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \frac{\left(1-\cos^2(x)\right)^3}{b\cos^2(x)+a}d\cos(x)$$

$$\Big\downarrow \text{300}$$

$$-\int \left(-\frac{\cos^4(x)}{b}+\frac{(a+3b)\cos^2(x)}{b^2}-\frac{a^2+3ba+3b^2}{b^3}+\frac{a^3+3ba^2+3b^2a+b^3}{b^3\left(b\cos^2(x)+a\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2+3ab+3b^2\right)\cos(x)}{b^3}-\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}$$

input
```
Int[Sin[x]^7/(a + b*Cos[x]^2),x]
```

output
```
-(((a + b)^3*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2))) + ((a^2
+ 3*a*b + 3*b^2)*Cos[x])/b^3 - ((a + 3*b)*Cos[x]^3)/(3*b^2) + Cos[x]^5/(5*
b)
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.10.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\left(\cos^{5}\left(x\right)\right) b^{2}}{5}-\frac{b \left(\cos^{3}\left(x\right)\right) a}{3}-b^{2} \left(\cos^{3}\left(x\right)\right)+a^{2} \cos\left(x\right)+3 b \cos\left(x\right) a+3 b^{2} \cos\left(x\right)}{b^{3}}+\frac{\left(-a^{3}-3 a^{2} b-3 a \,b^{2}-b^{3}\right) \arctan\left(\frac{b \cos\left(x\right)}{\sqrt{a b}}\right)}{b^{3} \sqrt{a b}}$
default	$\frac{\frac{\left(\cos^{5}\left(x\right)\right) b^{2}}{5}-\frac{b \left(\cos^{3}\left(x\right)\right) a}{3}-b^{2} \left(\cos^{3}\left(x\right)\right)+a^{2} \cos\left(x\right)+3 b \cos\left(x\right) a+3 b^{2} \cos\left(x\right)}{b^{3}}+\frac{\left(-a^{3}-3 a^{2} b-3 a \,b^{2}-b^{3}\right) \arctan\left(\frac{b \cos\left(x\right)}{\sqrt{a b}}\right)}{b^{3} \sqrt{a b}}$
risch	$\frac{{\mathrm e}^{i x} a^{2}}{2 b^{3}}+\frac{11 \,{\mathrm e}^{i x} a}{8 b^{2}}+\frac{19 \,{\mathrm e}^{i x}}{16 b}+\frac{{\mathrm e}^{-i x} a^{2}}{2 b^{3}}+\frac{11 \,{\mathrm e}^{-i x} a}{8 b^{2}}+\frac{19 \,{\mathrm e}^{-i x}}{16 b}+\frac{3 i \ln\left({\mathrm e}^{2 i x}-\frac{2 i a \,{\mathrm e}^{i x}}{\sqrt{a b}}+1\right) a}{2 \sqrt{a b}\, b}+\frac{3 i \ln\left({\mathrm e}^{2 i x}-\frac{2 i a \,{\mathrm e}^{i x}}{\sqrt{a b}}+\right.}{2 \sqrt{a b}\, b^{2}}$

input
```
int(sin(x)^7/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*cos(x)^5*b^2-1/3*b*cos(x)^3*a-b^2*cos(x)^3+a^2*cos(x)+3*b*cos(x
)*a+3*b^2*cos(x))+(-a^3-3*a^2*b-3*a*b^2-b^3)/b^3/(a*b)^(1/2)*arctan(b*cos(
x)/(a*b)^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.88

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{6\,ab^3\cos(x)^5 - 10\,(a^2b^2+3\,ab^3)\cos(x)^3 - 15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\cos(x)-}{b\cos(x)^2+a}\right.}{30\,ab^4}\right.$$

input
```
integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/30*(6*a*b^3*cos(x)^5 - 10*(a^2*b^2 + 3*a*b^3)*cos(x)^3 - 15*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos(x) - a
)/(b*cos(x)^2 + a)) + 30*(a^3*b + 3*a^2*b^2 + 3*a*b^3)*cos(x))/(a*b^4), 1/
15*(3*a*b^3*cos(x)^5 - 5*(a^2*b^2 + 3*a*b^3)*cos(x)^3 - 15*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a*b)*cos(x)/a) + 15*(a^3*b + 3*a^2*
b^2 + 3*a*b^3)*cos(x))/(a*b^4)]
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**7/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx = -\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2\cos{(x)}^5-5\,(ab+3\,b^2)\cos{(x)}^3+15\,(a^2+3\,ab+3\,b^2)\cos{(x)}}{15\,b^3}$$

input
```
integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/15*(3*b^2*cos(x)^5 - 5*(a*b + 3*b^2)*cos(x)^3 + 15*(a^2 + 3*a*b + 3*
b^2)*cos(x))/b^3
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx = -\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4\cos{(x)}^5-5\,ab^3\cos{(x)}^3-15\,b^4\cos{(x)}^3+15\,a^2b^2\cos{(x)}+45\,ab^3\cos{(x)}+45\,b^4\cos{(x)}}{15\,b^5}$$

input
```
integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/15*(3*b^4*cos(x)^5 - 5*a*b^3*cos(x)^3 - 15*b^4*cos(x)^3 + 15*a^2*b^2
*cos(x) + 45*a*b^3*cos(x) + 45*b^4*cos(x))/b^5
```

3.10.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx = \cos(x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}+\frac{3}{b}\right)}{b}\right)-\cos(x)^3\left(\frac{a}{3\,b^2}+\frac{1}{b}\right)+\frac{\cos(x)^5}{5\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\cos(x)\,(a+b)^3}{\sqrt{a}\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}\right)\,(a+b)^3}{\sqrt{a}\,b^{7/2}}$$

input
```
int(sin(x)^7/(a + b*cos(x)^2),x)
```

output
```
cos(x)*(3/b + (a*(a/b^2 + 3/b))/b) - cos(x)^3*(a/(3*b^2) + 1/b) + cos(x)^5
/(5*b) - (atan((b^(1/2)*cos(x)*(a + b)^3)/(a^(1/2)*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)))*(a + b)^3)/(a^(1/2)*b^(7/2))
```

3.11 $\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx$

3.11.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx=-\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}$$

output
```
(a+2*b)*cos(x)/b^2-1/3*cos(x)^3/b-(a+b)^2*arctan(cos(x)*b^(1/2)/a^(1/2))/b
^(5/2)/a^(1/2)
```

3.11.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(54) = 108.

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.15

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx=\frac{-\frac{12(a+b)^2\arctan\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{12(a+b)^2\arctan\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+3\sqrt{b}(4a+7b)\cos(x)-b^{3/2}\cos(3x)}{12b^{5/2}}$$

input
```
Integrate[Sin[x]^5/(a + b*Cos[x]^2),x]
```

output
```
((-12*(a + b)^2*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/Sqrt[a]
- (12*(a + b)^2*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/Sqrt[a]
+ 3*Sqrt[b]*(4*a + 7*b)*Cos[x] - b^(3/2)*Cos[3*x])/(12*b^(5/2))
```

3.11.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)^5}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)^5}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{ 3669}$$

$$-\int \frac{\left(1-\cos^2(x)\right)^2}{b\cos^2(x)+a}d\cos(x)$$

$$\Big\downarrow \text{ 300}$$

$$-\int \left(\frac{\cos^2(x)}{b}-\frac{a+2b}{b^2}+\frac{a^2+2ba+b^2}{b^2\left(b\cos^2(x)+a\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}$$

input
```
Int[Sin[x]^5/(a + b*Cos[x]^2),x]
```

output
```
-(((a + b)^2*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2))) + ((a +
2*b)*Cos[x])/b^2 - Cos[x]^3/(3*b)
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.11.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{b\left(\cos^3(x)\right)}{3}+\cos(x)a+2\cos(x)b}{b^2}+\frac{(-a^2-2ab-b^2)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
default	$\frac{-\frac{b\left(\cos^3(x)\right)}{3}+\cos(x)a+2\cos(x)b}{b^2}+\frac{(-a^2-2ab-b^2)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{e^{ix}a}{2b^2}+\frac{7e^{ix}}{8b}+\frac{e^{-ix}a}{2b^2}+\frac{7e^{-ix}}{8b}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a^2}{2\sqrt{ab}\,b^2}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a}{\sqrt{ab}\,b}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}$

input
```
int(sin(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/3*b*cos(x)^3+cos(x)*a+2*cos(x)*b)+(-a^2-2*a*b-b^2)/b^2/(a*b)^(1/
2)*arctan(b*cos(x)/(a*b)^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.81

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx = \left[-\frac{2\,ab^2\cos(x)^3+3\,(a^2+2\,ab+b^2)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\cos(x)-a}{b\cos(x)^2+a}\right)-6\,(a^2b+2\,ab^2)\cos(x)}{6\,ab^3},\ -\frac{ab^2\cos(x)^3+3\,(a^2+2\,ab+b^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\cos(x)}{a}\right)-3\,(a^2b+2\,ab^2)\cos(x)}{3\,ab^3}\right]$$

input
```
integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/6*(2*a*b^2*cos(x)^3 + 3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^
2 + 2*sqrt(-a*b)*cos(x) - a)/(b*cos(x)^2 + a)) - 6*(a^2*b + 2*a*b^2)*cos(x
))/(a*b^3), -1/3*(a*b^2*cos(x)^3 + 3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(
sqrt(a*b)*cos(x)/a) - 3*(a^2*b + 2*a*b^2)*cos(x))/(a*b^3)]
```

3.11.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**5/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx = -\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} - \frac{b\cos(x)^3-3\,(a+2\,b)\cos(x)}{3\,b^2}$$

input
```
integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-(a^2 + 2*a*b + b^2)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/3*(b*c
os(x)^3 - 3*(a + 2*b)*cos(x))/b^2
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx = -\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} - \frac{b^2\cos(x)^3-3\,ab\cos(x)-6\,b^2\cos(x)}{3\,b^3}$$

input
```
integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-(a^2 + 2*a*b + b^2)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/3*(b^2
*cos(x)^3 - 3*a*b*cos(x) - 6*b^2*cos(x))/b^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx = \cos(x)\left(\frac{a}{b^2}+\frac{2}{b}\right) - \frac{\cos(x)^3}{3\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\cos(x)\,(a+b)^2}{\sqrt{a}\,(a^2+2\,a\,b+b^2)}\right)\,(a+b)^2}{\sqrt{a}\,b^{5/2}}$$

input
```
int(sin(x)^5/(a + b*cos(x)^2),x)
```

output
```
cos(x)*(a/b^2 + 2/b) - cos(x)^3/(3*b) - (atan((b^(1/2)*cos(x)*(a + b)^2)/(
a^(1/2)*(2*a*b + a^2 + b^2)))*(a + b)^2)/(a^(1/2)*b^(5/2))
```

3.12 $\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx$

3.12.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx=-\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{\cos(x)}{b}$$

output
```
cos(x)/b-(a+b)*arctan(cos(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.12.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(36) = 72.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx =\frac{-\left((a+b)\arctan\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)\right)-(a+b)\arctan\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{b}\cos(x)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[Sin[x]^3/(a + b*Cos[x]^2),x]
```

output
```
(-((a + b)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]]) - (a + b)*Arc
Tan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*Sqrt[b]*Cos[x])/(S
qrt[a]*b^(3/2))
```

3.12.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3669, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)^3}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)^3}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \; 3669$$

$$-\int \frac{1-\cos^2(x)}{b\cos^2(x)+a}d\cos(x)$$

$$\Big\downarrow \; 299$$

$$\frac{\cos(x)}{b}-\frac{(a+b)\int \frac{1}{b\cos^2(x)+a}d\cos(x)}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{\cos(x)}{b}-\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Int[Sin[x]^3/(a + b*Cos[x]^2),x]
```

output
```
-(((a + b)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + Cos[x]/b
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.12.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\cos(x)}{b}+\frac{(-a-b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$
default	$\frac{\cos(x)}{b}+\frac{(-a-b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$
risch	$\frac{e^{ix}}{2b}+\frac{e^{-ix}}{2b}+\frac{i\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a}{2\sqrt{ab}\,b}+\frac{i\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a}{2\sqrt{ab}\,b}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right.}{2\sqrt{ab}}$

input
```
int(sin(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
cos(x)/b+(-a-b)/b/(a*b)^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx = \left[\frac{2\,ab\cos(x)-\sqrt{-ab}(a+b)\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\cos(x)-a}{b\cos(x)^2+a}\right)}{2\,ab^2}, \frac{ab\cos(x)-\sqrt{ab}(a+b)\arctan\left(\frac{\sqrt{ab}\cos(x)}{a}\right)}{ab^2}\right]$$

input
```
integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*cos(x) - sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*co
s(x) - a)/(b*cos(x)^2 + a)))/(a*b^2), (a*b*cos(x) - sqrt(a*b)*(a + b)*arct
an(sqrt(a*b)*cos(x)/a))/(a*b^2)]
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**3/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx = -\frac{(a+b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{\cos(x)}{b}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-(a + b)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b) + cos(x)/b
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx = -\frac{(a+b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{\cos(x)}{b}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-(a + b)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b) + cos(x)/b
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx = \frac{\cos(x)}{b} - \frac{\operatorname{atan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)(a+b)}{\sqrt{a}\,b^{3/2}}$$

input
```
int(sin(x)^3/(a + b*cos(x)^2),x)
```

output
```
cos(x)/b - (atan((b^(1/2)*cos(x))/a^(1/2))*(a + b))/(a^(1/2)*b^(3/2))
```

3.13 $\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx$

3.13.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
-arctan(cos(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[Sin[x]/(a + b*Cos[x]^2),x]
```

output
```
-(ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]))
```

3.13.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3669, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \frac{1}{b\cos^2(x)+a}d\cos(x)$$

$$\Big\downarrow \text{218}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[Sin[x]/(a + b*Cos[x]^2),x]
```

output
```
-(ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]))
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.13.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	18
default	$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	18
risch	$-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}+\frac{i\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}$	62

input
```
int(sin(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*b)^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.81

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\cos(x)-a}{b\cos(x)^2+a}\right)}{2\,ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\cos(x)}{a}\right)}{ab}\right]$$

input
```
integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos(x) - a)/(b*cos(x)^2 +
 a))/(a*b), -sqrt(a*b)*arctan(sqrt(a*b)*cos(x)/a)/(a*b)]
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\cos(x)} & \text{for } a=0 \wedge b=0 \\ -\frac{\cos(x)}{a} & \text{for } b=0 \\ \frac{1}{b\cos(x)} & \text{for } a=0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}}+\cos(x)\right)}{2b\sqrt{-\frac{a}{b}}} + \frac{\log\left(\sqrt{-\frac{a}{b}}+\cos(x)\right)}{2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a+b*cos(x)**2),x)
```

output
```
Piecewise((zoo/cos(x), Eq(a, 0) & Eq(b, 0)), (-cos(x)/a, Eq(b, 0)), (1/(b*
cos(x)), Eq(a, 0)), (-log(-sqrt(-a/b) + cos(x))/(2*b*sqrt(-a/b)) + log(sqr
t(-a/b) + cos(x))/(2*b*sqrt(-a/b)), True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-arctan(b*cos(x)/sqrt(a*b))/sqrt(a*b)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-arctan(b*cos(x)/sqrt(a*b))/sqrt(a*b)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx = -\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(sin(x)/(a + b*cos(x)^2),x)
```

output
```
-atan((b^(1/2)*cos(x))/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.14 $\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$

3.14.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}-\frac{\operatorname{arctanh}(\cos(x))}{a+b}$$

output
```
-arctanh(cos(x))/(a+b)-arctan(cos(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2
)
```

3.14.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx=\frac{-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}}+\log(1-\cos(x))-\log(1+\cos(x))}{2(a+b)}$$

input
```
Integrate[Csc[x]/(a + b*Cos[x]^2),x]
```

output
```
((-2*Sqrt[b]*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/Sqrt[a] + Log[1 - Cos[x]] -
 Log[1 + Cos[x]])/(2*(a + b))
```

3.14.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 25, 3669, 303, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{\cos\left(x+\frac{\pi}{2}\right)\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)}dx$$
$$\Big\downarrow \text{3669}$$
$$-\int \frac{1}{(1-\cos^2(x))\,(b\cos^2(x)+a)}d\cos(x)$$
$$\Big\downarrow \text{303}$$
$$-\frac{\int \frac{1}{1-\cos^2(x)}d\cos(x)}{a+b}-\frac{b\int \frac{1}{b\cos^2(x)+a}d\cos(x)}{a+b}$$
$$\Big\downarrow \text{218}$$
$$-\frac{\int \frac{1}{1-\cos^2(x)}d\cos(x)}{a+b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}-\frac{\text{arctanh}(\cos(x))}{a+b}$$

input
```
Int[Csc[x]/(a + b*Cos[x]^2),x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b))) - ArcTanh[
Cos[x]]/(a + b)
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.14.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\ln(1+\cos(x))}{2a+2b}-\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(\cos(x)-1)}{2a+2b}$	56
risch	$\frac{\ln(e^{ix}-1)}{a+b}-\frac{\ln(e^{ix}+1)}{a+b}+\frac{i\sqrt{ab}\,\ln\left(e^{2ix}-\frac{2i\sqrt{ab}\,e^{ix}}{b}+1\right)}{2a(a+b)}-\frac{i\sqrt{ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{ab}\,e^{ix}}{b}+1\right)}{2a(a+b)}$	111

input
```
int(csc(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(2*a+2*b)*ln(1+cos(x))-b/(a+b)/(a*b)^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))
+1/(2*a+2*b)*ln(cos(x)-1)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.69

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right)-\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a+b)},\right.$$
$$\left.-\frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\cos(x)\right)+\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a+b)}\right]$$

input
```
integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x) - a)/(b*cos(x)^2
+ a)) - log(1/2*cos(x) + 1/2) + log(-1/2*cos(x) + 1/2))/(a + b), -1/2*(2*s
qrt(b/a)*arctan(sqrt(b/a)*cos(x)) + log(1/2*cos(x) + 1/2) - log(-1/2*cos(x
) + 1/2))/(a + b)]
```

3.14.6 Sympy [F]

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx = \int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(csc(x)/(a+b*cos(x)**2),x)
```

output
```
Integral(csc(x)/(a + b*cos(x)**2), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx = -\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a+b\right)} + \frac{\log\left(\cos\left(x\right)-1\right)}{2\left(a+b\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-b*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*log(cos(x) + 1)/(a
 + b) + 1/2*log(cos(x) - 1)/(a + b)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx = -\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a+b\right)} + \frac{\log\left(-\cos\left(x\right)+1\right)}{2\left(a+b\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-b*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*log(cos(x) + 1)/(a
 + b) + 1/2*log(-cos(x) + 1)/(a + b)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 853, normalized size of antiderivative = 20.31

$$\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}+4\,b^3\,\cos(x)\right)1\mathrm{i}}{2\,(a+b)}-\frac{\left(\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}-4\,b \cdots\right.}{2\,(a+b)}}{\frac{\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}+4\,b^3\,\cos(x)}{2\,(a+b)}+\frac{\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}-4\,b^3 \cdots}{2\,(a+b)}}\right.}{a+b}$$

$$+\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\cos(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)1\mathrm{i}}{a^2+b\,a}+\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b \cdots\right.}{}\right.}{}}{\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\cos(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)}{a^2+b\,a}-\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b \cdots\right.}{}\right.}{}}\right.}{a\,(a+b)}$$

input
```
int(1/(sin(x)*(a + b*cos(x)^2)),x)
```

output
```
(atan(((((8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b
^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)) + 4*b^3*cos(x))*1i)/(2*(a + b))
- (((8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 -
8*a^3*b^2))/(2*(a + b)))/(2*(a + b)) - 4*b^3*cos(x))*1i)/(2*(a + b)))/(((8
*a*b^3 + 4*b^4 + 4*a^2*b^2 - (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*
b^2))/(2*(a + b)))/(2*(a + b)) + 4*b^3*cos(x))/(2*(a + b)) + ((8*a*b^3 + 4
*b^4 + 4*a^2*b^2 + (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(
a + b)))/(2*(a + b)) - 4*b^3*cos(x))/(2*(a + b))))*1i)/(a + b) + (atan((((
-a*b)^(1/2)*(2*b^3*cos(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (
cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a
^2))))/(2*(a*b + a^2)))*1i)/(a*b + a^2) + ((-a*b)^(1/2)*(2*b^3*cos(x) - ((
-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (cos(x)*(-a*b)^(1/2)*(8*a*b^4 +
 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*1i)/(a
*b + a^2))/(((-a*b)^(1/2)*(2*b^3*cos(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 +
 2*a^2*b^2 - (cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2
))/(4*(a*b + a^2))))/(2*(a*b + a^2))))/(a*b + a^2) - ((-a*b)^(1/2)*(2*b^3*
cos(x) - ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (cos(x)*(-a*b)^(1/2)
*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^
2))))/(a*b + a^2)))*(-a*b)^(1/2)*1i)/(a*(a + b))
```

3.14. $\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$

3.15 $\int \frac{\csc^3(x)}{a+b\cos^2(x)} dx$

3.15.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2} - \frac{(a+3b)\text{arctanh}(\cos(x))}{2(a+b)^2} - \frac{\cot(x)\csc(x)}{2(a+b)}$$

output
```
-1/2*(a+3*b)*arctanh(cos(x))/(a+b)^2-1/2*cot(x)*csc(x)/(a+b)-b^(3/2)*arcta
n(cos(x)*b^(1/2)/a^(1/2))/(a+b)^2/a^(1/2)
```

3.15.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.26

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx = \frac{-8b^{3/2}\arctan\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right) - 8b^{3/2}\arctan\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right) + \sqrt{a}\left(-\left((a+b)\csc^2\left(\frac{x}{2}\right)\right) - 4(a+3b)\left(\text{lo}\right.\right.}{8\sqrt{a}(a+b)^2}$$

input
```
Integrate[Csc[x]^3/(a + b*Cos[x]^2),x]
```

output
```
(-8*b^(3/2)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]] - 8*b^(3/2)*A
rcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*(-((a + b)*Csc[x
/2]^2) - 4*(a + 3*b)*(Log[Cos[x/2]] - Log[Sin[x/2]]) + (a + b)*Sec[x/2]^2)
)/(8*Sqrt[a]*(a + b)^2)
```

3.15.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 25, 3669, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(x+\frac{\pi}{2}\right)^3\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^3\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \frac{1}{(1-\cos^2(x))^2\,(b\cos^2(x)+a)}d\cos(x)$$

$$\Big\downarrow \text{316}$$

$$-\frac{\int \frac{b\cos^2(x)+a+2b}{(1-\cos^2(x))(b\cos^2(x)+a)}d\cos(x)}{2(a+b)}-\frac{\cos(x)}{2(a+b)\,(1-\cos^2(x))}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{2b^2\int \frac{1}{b\cos^2(x)+a}d\cos(x)}{a+b}+\frac{(a+3b)\int \frac{1}{1-\cos^2(x)}d\cos(x)}{a+b}}{2(a+b)}-\frac{\cos(x)}{2(a+b)\,(1-\cos^2(x))}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{(a+3b)\int \frac{1}{1-\cos^2(x)}d\cos(x)}{a+b}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}-\frac{\cos(x)}{2(a+b)\left(1-\cos^2(x)\right)}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{(a+3b)\text{arctanh}(\cos(x))}{a+b}}{2(a+b)}-\frac{\cos(x)}{2(a+b)\left(1-\cos^2(x)\right)}$$

input
```
Int[Csc[x]^3/(a + b*Cos[x]^2),x]
```

output
```
-1/2*((2*b^(3/2)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((a
 + 3*b)*ArcTanh[Cos[x]])/(a + b))/(a + b) - Cos[x]/(2*(a + b)*(1 - Cos[x]^
2))
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.15.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

method	result
default	$\frac{1}{(4a+4b)(1+\cos(x))}+\frac{(-a-3b)\ln(1+\cos(x))}{4(a+b)^2}-\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(\cos(x)-1)}+\frac{(a+3b)\ln(\cos(x)-1)}{4(a+b)^2}$
risch	$\frac{e^{3ix}+e^{ix}}{(e^{2ix}-1)^2(a+b)}+\frac{\ln(e^{ix}-1)a}{2a^2+4ab+2b^2}+\frac{3\ln(e^{ix}-1)b}{2(a^2+2ab+b^2)}-\frac{\ln(e^{ix}+1)a}{2(a^2+2ab+b^2)}-\frac{3\ln(e^{ix}+1)b}{2(a^2+2ab+b^2)}-\frac{i\sqrt{ab}\,b\ln\left(e^{2ix}+\frac{2i\sqrt{ab}\,e^{ix}}{b}+1\right)}{2a(a+b)^2}+$

input
```
int(csc(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(4*a+4*b)/(1+cos(x))+1/4/(a+b)^2*(-a-3*b)*ln(1+cos(x))-b^2/(a+b)^2/(a*b)
^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))+1/(4*a+4*b)/(cos(x)-1)+1/4*(a+3*b)/(a+
b)^2*ln(cos(x)-1)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(50) = 100$.

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.42

$$\int \frac{\csc^3(x)}{a + b\cos^2(x)}\,dx$$
$$= \left[\frac{2\left(b\cos\left(x\right)^2 - b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2 - 2\,a\sqrt{-\frac{b}{a}}\cos(x) - a}{b\cos(x)^2 + a}\right) + 2\left(a + b\right)\cos\left(x\right) - \left(\left(a + 3\,b\right)\cos\left(x\right)^2 - a - 3\,b\right)\mathrm{lo}}{4\left(\left(a^2 + 2\,ab + b^2\right)\cos\left(x\right)^2 - a^2 - 2\,ab}\right.$$
$$- \frac{4\left(b\cos\left(x\right)^2 - b\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\cos\left(x\right)\right) - 2\left(a + b\right)\cos\left(x\right) + \left(\left(a + 3\,b\right)\cos\left(x\right)^2 - a - 3\,b\right)\log\left(\frac{1}{2}\cos\right.}{4\left(\left(a^2 + 2\,ab + b^2\right)\cos\left(x\right)^2 - a^2 - 2\,ab - b\right.}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(b*cos(x)^2 - b)*sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x
) - a)/(b*cos(x)^2 + a)) + 2*(a + b)*cos(x) - ((a + 3*b)*cos(x)^2 - a - 3*
b)*log(1/2*cos(x) + 1/2) + ((a + 3*b)*cos(x)^2 - a - 3*b)*log(-1/2*cos(x)
+ 1/2))/((a^2 + 2*a*b + b^2)*cos(x)^2 - a^2 - 2*a*b - b^2), -1/4*(4*(b*cos
(x)^2 - b)*sqrt(b/a)*arctan(sqrt(b/a)*cos(x)) - 2*(a + b)*cos(x) + ((a + 3
*b)*cos(x)^2 - a - 3*b)*log(1/2*cos(x) + 1/2) - ((a + 3*b)*cos(x)^2 - a -
3*b)*log(-1/2*cos(x) + 1/2))/((a^2 + 2*a*b + b^2)*cos(x)^2 - a^2 - 2*a*b -
 b^2)]
```

3.15.6 Sympy [F]

$$\int \frac{\csc^3(x)}{a + b\cos^2(x)}\,dx = \int \frac{\csc^3\left(x\right)}{a + b\cos^2\left(x\right)}\,dx$$

input
```
integrate(csc(x)**3/(a+b*cos(x)**2),x)
```

output
```
Integral(csc(x)**3/(a + b*cos(x)**2), x)
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(50) = 100$.

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx = -\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{(a+3\,b)\log\left(\cos\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} + \frac{(a+3\,b)\log\left(\cos\left(x\right)-1\right)}{4\,(a^2+2\,ab+b^2)} + \frac{\cos\left(x\right)}{2\left((a+b)\cos\left(x\right)^2-a-b\right)}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-b^2*arctan(b*cos(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 1/4*(a +
 3*b)*log(cos(x) + 1)/(a^2 + 2*a*b + b^2) + 1/4*(a + 3*b)*log(cos(x) - 1)/
(a^2 + 2*a*b + b^2) + 1/2*cos(x)/((a + b)*cos(x)^2 - a - b)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(50) = 100$.

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.66

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx = -\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{(a+3\,b)\log\left(\cos\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} + \frac{(a+3\,b)\log\left(-\cos\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} + \frac{\cos\left(x\right)}{2\left(\cos\left(x\right)^2-1\right)(a+b)}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-b^2*arctan(b*cos(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 1/4*(a +
 3*b)*log(cos(x) + 1)/(a^2 + 2*a*b + b^2) + 1/4*(a + 3*b)*log(-cos(x) + 1)
/(a^2 + 2*a*b + b^2) + 1/2*cos(x)/((cos(x)^2 - 1)*(a + b))
```

3.15.9 Mupad [B] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 1138, normalized size of antiderivative = 18.35

$$\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx$$
$$= \ln(\cos(x)-1)\left(\frac{b}{2(a+b)^2}+\frac{1}{4(a+b)}\right)-\frac{\cos(x)}{2\sin(x)^2(a+b)}-\frac{\ln(\cos(x)+1)(a+3b)}{4(a+b)^2}$$
$$-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{-a\,b^3}\left(\frac{\cos(x)\left(a^2b^3+6ab^4+13b^5\right)}{4\left(a^2+2ab+b^2\right)}+\frac{\left(\frac{2a^5b^2+12a^4b^3+28a^3b^4+32a^2b^5+18ab^6+4b^7}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\cos(x)\sqrt{-a\,b^3}\left(-16a^5b^2-48a^4b^3-32a^3\ldots\right)}{8\left(a^2+2ab+b^2\right)\left(a^3+2\ldots\right)}\right.}{2\left(a^3+2a^2b+ab^2\right)}}{a^3+2a^2b+ab^2}}{\frac{\frac{3b^5}{2}+\frac{ab^4}{2}}{a^3+3a^2b+3ab^2+b^3}-\frac{\sqrt{-a\,b^3}\left(\frac{\cos(x)\left(a^2b^3+6ab^4+13b^5\right)}{4\left(a^2+2ab+b^2\right)}+\frac{\left(\frac{2a^5b^2+12a^4b^3+28a^3b^4+32a^2b^5+18ab^6+4b^7}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\cos(x)\sqrt{-a\,b^3}\left(-16a^5b^2-\ldots\right)}{8\left(a^2+2a\ldots\right)}\right.}{2\left(a^3+2a^2b+ab^2\right)}}{a^3+2a^2b+ab^2}}\right)}{\ldots}$$

input
```
int(1/(sin(x)^3*(a + b*cos(x)^2)),x)
```

output
```
log(cos(x) - 1)*(b/(2*(a + b)^2) + 1/(4*(a + b))) - cos(x)/(2*sin(x)^2*(a
+ b)) - (log(cos(x) + 1)*(a + 3*b))/(4*(a + b)^2) - (atan((((-a*b^3)^(1/2)
*((cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*
b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^
2 + 3*a^2*b + a^3 + b^3)) - (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32
*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(
a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/
(a*b^2 + 2*a^2*b + a^3) + ((-a*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 13*b^5 + a^2
*b^3))/(4*(2*a*b + a^2 + b^2)) - (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3
*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (cos(
x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^
3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)
^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2 + 2*a^2*b + a^3))/(((a*b^4
)/2 + (3*b^5)/2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - ((-a*b^3)^(1/2)*((cos(x
)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6 + 4*
b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^
2*b + a^3 + b^3)) - (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5
 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 +
2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))))/(a*b^2 + 2*
a^2*b + a^3) + ((-a*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(...
```

3.16 $\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$

3.16.1 Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3} - \frac{(3a^2+10ab+15b^2)\operatorname{arctanh}(\cos(x))}{8(a+b)^3} - \frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2} - \frac{\cot(x)\csc^3(x)}{4(a+b)}$$

output
```
-1/8*(3*a^2+10*a*b+15*b^2)*arctanh(cos(x))/(a+b)^3-1/8*(3*a+7*b)*cot(x)*cs
c(x)/(a+b)^2-1/4*cot(x)*csc(x)^3/(a+b)-b^(5/2)*arctan(cos(x)*b^(1/2)/a^(1/
2))/(a+b)^3/a^(1/2)
```

3.16.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 204 vs. 2(94) = 188.

Time = 1.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.17

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx = \frac{-64b^{5/2}\arctan\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right) - 64b^{5/2}\arctan\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right) + \sqrt{a}\left(-2(3a^2+10ab+7b^2)\csc^2\left(\frac{x}{2}\right) - \right.}{}$$

input
```
Integrate[Csc[x]^5/(a + b*Cos[x]^2),x]
```

output
```
(-64*b^(5/2)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]] - 64*b^(5/2)
*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*(-2*(3*a^2 + 1
0*a*b + 7*b^2)*Csc[x/2]^2 - (a + b)^2*Csc[x/2]^4 - 8*(3*a^2 + 10*a*b + 15*
b^2)*(Log[Cos[x/2]] - Log[Sin[x/2]]) + 2*(3*a^2 + 10*a*b + 7*b^2)*Sec[x/2]
^2 + (a + b)^2*Sec[x/2]^4))/(64*Sqrt[a]*(a + b)^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 25, 3669, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{\cos\left(x+\frac{\pi}{2}\right)^5\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^5\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)}dx$$

$$\Big\downarrow \ 3669$$

$$-\int \frac{1}{(1-\cos^2(x))^3\left(b\cos^2(x)+a\right)}d\cos(x)$$

$$\Big\downarrow \ 316$$

$$-\frac{\int \frac{3b\cos^2(x)+3a+4b}{(1-\cos^2(x))^2(b\cos^2(x)+a)}d\cos(x)}{4(a+b)}-\frac{\cos(x)}{4(a+b)\left(1-\cos^2(x)\right)^2}$$

$$\Big\downarrow \ 402$$

$$-\frac{\frac{\int \frac{3a^2+7ba+8b^2+b(3a+7b)\cos^2(x)}{\left(1-\cos^2(x)\right)\left(b\cos^2(x)+a\right)}d\cos(x)}{2(a+b)}+\frac{(3a+7b)\cos(x)}{2(a+b)(1-\cos^2(x))}}{4(a+b)}-\frac{\cos(x)}{4(a+b)\left(1-\cos^2(x)\right)^2}$$

$$\Big\downarrow \ 397$$

$$-\frac{\frac{\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\cos^2(x)}d\cos(x)}{a+b}+\frac{8b^3\int \frac{1}{b\cos^2(x)+a}d\cos(x)}{a+b}}{2(a+b)}+\frac{(3a+7b)\cos(x)}{2(a+b)(1-\cos^2(x))}}{4(a+b)}-\frac{\cos(x)}{4(a+b)\left(1-\cos^2(x)\right)^2}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\cos^2(x)}d\cos(x)}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\cos(x)}{2(a+b)(1-\cos^2(x))}}{4(a+b)}-\frac{\cos(x)}{4(a+b)\left(1-\cos^2(x)\right)^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\frac{\left(3a^2+10ab+15b^2\right)\text{arctanh}(\cos(x))}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\cos(x)}{2(a+b)(1-\cos^2(x))}}{4(a+b)}-\frac{\cos(x)}{4(a+b)\left(1-\cos^2(x)\right)^2}$$

input
```
Int[Csc[x]^5/(a + b*Cos[x]^2),x]
```

output
```
-1/4*Cos[x]/((a + b)*(1 - Cos[x]^2)^2) - (((8*b^(5/2)*ArcTan[(Sqrt[b]*Cos[
x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[Cos[x
]])/(a + b))/(2*(a + b)) + ((3*a + 7*b)*Cos[x])/(2*(a + b)*(1 - Cos[x]^2))
)/(4*(a + b))
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.16.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

method	result
default	$\frac{1}{2(8a+8b)(1+\cos(x))^2}-\frac{-3a-7b}{16(a+b)^2(1+\cos(x))}+\frac{(-3a^2-10ab-15b^2)\ln(1+\cos(x))}{16(a+b)^3}-\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}-\frac{1}{2(8a+8b)(\cos(x)-}$
risch	$\frac{3a\,e^{7ix}+7b\,e^{7ix}-11a\,e^{5ix}-15b\,e^{5ix}-11a\,e^{3ix}-15b\,e^{3ix}+3\,e^{ix}a+7\,e^{ix}b}{4(a+b)^2(e^{2ix}-1)^4}-\frac{3\ln(e^{ix}+1)a^2}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{5\ln(e^{ix}+1)ab}{4(a^3+3a^2b+3a\,b^2+b^3)}-\frac{}{8(}$

input
```
int(csc(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(8*a+8*b)/(1+cos(x))^2-1/16*(-3*a-7*b)/(a+b)^2/(1+cos(x))+1/16/(a+b)^3
*(-3*a^2-10*a*b-15*b^2)*ln(1+cos(x))-b^3/(a+b)^3/(a*b)^(1/2)*arctan(b*cos(
x)/(a*b)^(1/2))-1/2/(8*a+8*b)/(cos(x)-1)^2-1/16*(-3*a-7*b)/(a+b)^2/(cos(x)
-1)+1/16*(3*a^2+10*a*b+15*b^2)/(a+b)^3*ln(cos(x)-1)
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(80) = 160$.

Time = 0.32 (sec) , antiderivative size = 592, normalized size of antiderivative = 6.30

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{2\left(3a^2+10ab+7b^2\right)\cos(x)^3+8\left(b^2\cos(x)^4-2b^2\cos(x)^2+b^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right)-}{}\right.$$

input
```
integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a^2 + 10*a*b + 7*b^2)*cos(x)^3 + 8*(b^2*cos(x)^4 - 2*b^2*cos(x
)^2 + b^2)*sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x) - a)/(b*cos(
x)^2 + a)) - 2*(5*a^2 + 14*a*b + 9*b^2)*cos(x) - ((3*a^2 + 10*a*b + 15*b^2
)*cos(x)^4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^
2)*log(1/2*cos(x) + 1/2) + ((3*a^2 + 10*a*b + 15*b^2)*cos(x)^4 - 2*(3*a^2
+ 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*log(-1/2*cos(x) + 1
/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
 b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2), 1/16*(2*(3*a^2 + 10*a*
b + 7*b^2)*cos(x)^3 - 16*(b^2*cos(x)^4 - 2*b^2*cos(x)^2 + b^2)*sqrt(b/a)*a
rctan(sqrt(b/a)*cos(x)) - 2*(5*a^2 + 14*a*b + 9*b^2)*cos(x) - ((3*a^2 + 10
*a*b + 15*b^2)*cos(x)^4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 1
0*a*b + 15*b^2)*log(1/2*cos(x) + 1/2) + ((3*a^2 + 10*a*b + 15*b^2)*cos(x)^
4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*log(-1
/2*cos(x) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^4 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2)]
```

3.16.6 Sympy [F]

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx = \int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(csc(x)**5/(a+b*cos(x)**2),x)
```

output
```
Integral(csc(x)**5/(a + b*cos(x)**2), x)
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(80) = 160.

Time = 0.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.13

$$\begin{aligned}&\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx\\ &= -\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} - \frac{(3a^2+10ab+15b^2)\log(\cos(x)+1)}{16(a^3+3a^2b+3ab^2+b^3)}\\ &\quad+\frac{(3a^2+10ab+15b^2)\log(\cos(x)-1)}{16(a^3+3a^2b+3ab^2+b^3)}\\ &\quad+\frac{(3a+7b)\cos(x)^3-(5a+9b)\cos(x)}{8\left((a^2+2ab+b^2)\cos(x)^4-2(a^2+2ab+b^2)\cos(x)^2+a^2+2ab+b^2\right)}\end{aligned}$$

input
```
integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-b^3*arctan(b*cos(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)
) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) - 1)/(a^3 + 3*a^2*b +
 3*a*b^2 + b^3) + 1/8*((3*a + 7*b)*cos(x)^3 - (5*a + 9*b)*cos(x))/((a^2 +
2*a*b + b^2)*cos(x)^4 - 2*(a^2 + 2*a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2
)
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(80) = 160$.

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx = -\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} - \frac{(3a^2+10ab+15b^2)\log(\cos(x)+1)}{16(a^3+3a^2b+3ab^2+b^3)} + \frac{(3a^2+10ab+15b^2)\log(-\cos(x)+1)}{16(a^3+3a^2b+3ab^2+b^3)} + \frac{3a\cos(x)^3+7b\cos(x)^3-5a\cos(x)-9b\cos(x)}{8(a^2+2ab+b^2)\left(\cos(x)^2-1\right)^2}$$

input
```
integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-b^3*arctan(b*cos(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)
) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-cos(x) + 1)/(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3) + 1/8*(3*a*cos(x)^3 + 7*b*cos(x)^3 - 5*a*cos(x) - 9*b*cos
(x))/((a^2 + 2*a*b + b^2)*(cos(x)^2 - 1)^2)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 833, normalized size of antiderivative = 8.86

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx =$$

$$-\frac{3a^3\,\mathrm{atanh}(\cos(x)) - 3a^3\cos(x)^3 + 5a^3\cos(x) + 9ab^2\cos(x) + 14a^2b\cos(x) - 6a^3\,\mathrm{atanh}(\cos(x))}{}$$

input
```
int(1/(sin(x)^5*(a + b*cos(x)^2)),x)
```

output
```
-(atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a*b^5)^(3/2)*64i + a^6*b*
cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*co
s(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*co
s(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5
*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*(-a*b^5)^(1/2)*8i - 3*a^3*cos(x)^3 + 3*a^3
*atanh(cos(x)) + 5*a^3*cos(x) - atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(
x)*(-a*b^5)^(3/2)*64i + a^6*b*cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-
a*b^5)^(1/2)*289i + a^3*b^4*cos(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-
a*b^5)^(1/2)*190i + a^5*b^2*cos(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a
^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*cos(x)^2*(-a
*b^5)^(1/2)*16i + atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a*b^5)^(3
/2)*64i + a^6*b*cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^(1/2)*2
89i + a^3*b^4*cos(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(1/2)*1
90i + a^5*b^2*cos(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a
^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*cos(x)^4*(-a*b^5)^(1/2)*8i
 + 9*a*b^2*cos(x) + 14*a^2*b*cos(x) - 6*a^3*atanh(cos(x))*cos(x)^2 + 3*a^3
*atanh(cos(x))*cos(x)^4 - 7*a*b^2*cos(x)^3 - 10*a^2*b*cos(x)^3 + 15*a*b^2*
atanh(cos(x)) + 10*a^2*b*atanh(cos(x)) - 30*a*b^2*atanh(cos(x))*cos(x)^2 -
 20*a^2*b*atanh(cos(x))*cos(x)^2 + 15*a*b^2*atanh(cos(x))*cos(x)^4 + 10*a^
2*b*atanh(cos(x))*cos(x)^4)/(8*a^4*cos(x)^4 - 16*a^4*cos(x)^2 + 8*a*b^3...
```

3.16. $\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$

3.17 $\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx$

3.17.1 Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = -\frac{(8a^2+20ab+15b^2)\,x}{8b^3} - \frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3} + \frac{(4a+7b)\cos(x)\sin(x)}{8b^2} + \frac{\cos(x)\sin^3(x)}{4b}$$

output
```
-1/8*(8*a^2+20*a*b+15*b^2)*x/b^3+1/8*(4*a+7*b)*cos(x)*sin(x)/b^2+1/4*cos(x
)*sin(x)^3/b-(a+b)^(5/2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/b^3/a^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = \frac{-4(8a^2+20ab+15b^2)\,x + \frac{32(a+b)^{5/2}\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 8b(a+2b)\sin(2x) - b^2\sin(4x)}{32b^3}$$

input
```
Integrate[Sin[x]^6/(a + b*Cos[x]^2),x]
```

output
```
(-4*(8*a^2 + 20*a*b + 15*b^2)*x + (32*(a + b)^(5/2)*ArcTan[(Sqrt[a]*Tan[x]
)/Sqrt[a + b]])/Sqrt[a] + 8*b*(a + 2*b)*Sin[2*x] - b^2*Sin[4*x])/(32*b^3)
```

3.17.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3670, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^6}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$-\int \frac{1}{\left(\cot^2(x)+1\right)^3\left((a+b)\cot^2(x)+a\right)}d\cot(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\cot(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\int \frac{-3(a+b)\cot^2(x)+a+4b}{\left(\cot^2(x)+1\right)^2\left((a+b)\cot^2(x)+a\right)}d\cot(x)}{4b}$$

$$\Big\downarrow \text{402}$$

$$\frac{\cot(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{\int \frac{4a^2+9ba+8b^2-(a+b)(4a+7b)\cot^2(x)}{\left(\cot^2(x)+1\right)\left((a+b)\cot^2(x)+a\right)}d\cot(x)}{2b}-\frac{(4a+7b)\cot(x)}{2b(\cot^2(x)+1)}}{4b}$$

$$\Big\downarrow \text{397}$$

$$\frac{\cot(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{\frac{8(a+b)^3\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{\left(8a^2+20ab+15b^2\right)\int \frac{1}{\cot^2(x)+1}d\cot(x)}{b}}{2b}-\frac{(4a+7b)\cot(x)}{2b(\cot^2(x)+1)}}{4b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\cot(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{\frac{8(a+b)^3\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{\left(8a^2+20ab+15b^2\right)\arctan(\cot(x))}{b}}{2b}-\frac{(4a+7b)\cot(x)}{2b(\cot^2(x)+1)}}{4b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\cot(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{\frac{8(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\left(8a^2+20ab+15b^2\right)\arctan(\cot(x))}{b}}{2b}-\frac{(4a+7b)\cot(x)}{2b(\cot^2(x)+1)}}{4b}$$

input
```
Int[Sin[x]^6/(a + b*Cos[x]^2),x]
```

output
```
Cot[x]/(4*b*(1 + Cot[x]^2)^2) - ((-(((8*a^2 + 20*a*b + 15*b^2)*ArcTan[Cot[
x]])/b) + (8*(a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*
b))/(2*b) - ((4*a + 7*b)*Cot[x])/(2*b*(1 + Cot[x]^2)))/(4*b)
```

3.17.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.17.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

method	result
default	$\frac{(a+b)^3 \arctan\left(\frac{a \tan(x)}{\sqrt{(a+b)a}}\right)}{b^3 \sqrt{(a+b)a}} - \frac{\frac{\left(-\frac{1}{2}ab-\frac{9}{8}b^2\right)\left(\tan^3(x)\right)+\left(-\frac{1}{2}ab-\frac{7}{8}b^2\right)\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan(\tan(x))}{8}}{b^3}$
risch	$-\frac{x\,a^2}{b^3}-\frac{5xa}{2b^2}-\frac{15x}{8b}-\frac{i e^{2ix}a}{8b^2}-\frac{i e^{2ix}}{4b}+\frac{i e^{-2ix}a}{8b^2}+\frac{i e^{-2ix}}{4b}-\frac{a\sqrt{-(a+b)a}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-(a+b)a}+2a+b}{b}\right)}{2b^3}-\frac{\sqrt{-(a+b)a}}{}$

input
```
int(sin(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(a+b)^3/b^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))-1/b^3*(((-1/2
*a*b-9/8*b^2)*tan(x)^3+(-1/2*a*b-7/8*b^2)*tan(x))/(tan(x)^2+1)^2+1/8*(8*a^
2+20*a*b+15*b^2)*arctan(tan(x)))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.24

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{2\,(a^2+2\,ab+b^2)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right.}{8\,b^3}\right.$$
$$-\frac{4\,(a^2+2\,ab+b^2)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)+(8\,a^2+20\,ab+15\,b^2)x+\left(2\,b^2\cos(x)^3-(4\,a\right.}{8\,b^3}$$

input
```
integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*co
s(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*cos(
x))*sqrt(-(a + b)/a)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))
- (8*a^2 + 20*a*b + 15*b^2)*x - (2*b^2*cos(x)^3 - (4*a*b + 9*b^2)*cos(x))*
sin(x))/b^3, -1/8*(4*(a^2 + 2*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a
+ b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*
a*b + 15*b^2)*x + (2*b^2*cos(x)^3 - (4*a*b + 9*b^2)*cos(x))*sin(x))/b^3]
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**6/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = \frac{(4\,a+9\,b)\tan\left(x\right)^3+(4\,a+7\,b)\tan\left(x\right)}{8\left(b^2\tan\left(x\right)^4+2\,b^2\tan\left(x\right)^2+b^2\right)} - \frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3} + \frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^3}$$

input
```
integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
1/8*((4*a + 9*b)*tan(x)^3 + (4*a + 7*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(
x)^2 + b^2) - 1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = -\frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3} + \frac{(a^3+3\,a^2b+3\,ab^2+b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}b^3} + \frac{4\,a\tan\left(x\right)^3+9\,b\tan\left(x\right)^3+4\,a\tan\left(x\right)+7\,b\tan\left(x\right)}{8\left(\tan\left(x\right)^2+1\right)^2b^2}$$

input
```
integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi
*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/(sqrt(a^2 +
a*b)*b^3) + 1/8*(4*a*tan(x)^3 + 9*b*tan(x)^3 + 4*a*tan(x) + 7*b*tan(x))/((
tan(x)^2 + 1)^2*b^2)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 681, normalized size of antiderivative = 7.74

$$\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx = \frac{\frac{\tan(x)^3\,(4\,a+9\,b)}{8\,b^2}+\frac{\tan(x)\,(4\,a+7\,b)}{8\,b^2}}{\tan{(x)}^4+2\tan{(x)}^2+1} - \frac{\operatorname{atanh}\left(\frac{95\,a^2\tan(x)\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3\,b^3-5\,a^2\,b^4-a\,b^5}}{32\left(2\,a\,b^4+\frac{469\,a^4\,b}{32}+\frac{215\,a^5}{32}+\frac{287\,a^2\,b^3}{32}+\frac{517\,a^3\,b^2}{32}+\frac{5\,a^6}{4\,b}\right)}+\frac{5\,a^3\tan(x)\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3\,b^3-5\,a^2\,b^4-a\,b^5}}{4\left(\frac{5\,a^6}{4}+\frac{215\,a^5\,b}{32}+\frac{469\,a^4\,b^2}{32}+\frac{517\,a^3\,b^3}{32}+\frac{287\,a^2\,b^4}{32}+2\,a\,b^5\right)}+\frac{2\,a\,\mathrm{ta}}{}\right.}{a\,b^3} + \frac{\operatorname{atan}\left(\frac{5717\,a^3\tan(x)}{256\left(\frac{15\,a\,b^2}{4}+\frac{3665\,a^2\,b}{256}+\frac{5717\,a^3}{256}+\frac{1143\,a^4}{64\,b}+\frac{235\,a^5}{32\,b^2}+\frac{5\,a^6}{4\,b^3}\right)}+\frac{3665\,a^2\tan(x)}{256\left(\frac{15\,a\,b}{4}+\frac{3665\,a^2}{256}+\frac{5717\,a^3}{256\,b}+\frac{1143\,a^4}{64\,b^2}+\frac{235\,a^5}{32\,b^3}+\frac{5\,a^6}{4\,b^4}\right)}+\frac{}{64\left(\frac{15\,a\,b^3}{4}+\right.}\right.}{}$$

input
```
int(sin(x)^6/(a + b*cos(x)^2),x)
```

output
```
((tan(x)^3*(4*a + 9*b))/(8*b^2) + (tan(x)*(4*a + 7*b))/(8*b^2))/(2*tan(x)^
2 + tan(x)^4 + 1) + (atan((5717*a^3*tan(x))/(256*((15*a*b^2)/4 + (3665*a^2
*b)/256 + (5717*a^3)/256 + (1143*a^4)/(64*b) + (235*a^5)/(32*b^2) + (5*a^6
)/(4*b^3))) + (3665*a^2*tan(x))/(256*((15*a*b)/4 + (3665*a^2)/256 + (5717*
a^3)/(256*b) + (1143*a^4)/(64*b^2) + (235*a^5)/(32*b^3) + (5*a^6)/(4*b^4))
) + (1143*a^4*tan(x))/(64*((15*a*b^3)/4 + (5717*a^3*b)/256 + (1143*a^4)/64
 + (3665*a^2*b^2)/256 + (235*a^5)/(32*b) + (5*a^6)/(4*b^2))) + (235*a^5*ta
n(x))/(32*((15*a*b^4)/4 + (1143*a^4*b)/64 + (235*a^5)/32 + (3665*a^2*b^3)/
256 + (5717*a^3*b^2)/256 + (5*a^6)/(4*b))) + (5*a^6*tan(x))/(4*((15*a*b^5)
/4 + (235*a^5*b)/32 + (5*a^6)/4 + (3665*a^2*b^4)/256 + (5717*a^3*b^3)/256
+ (1143*a^4*b^2)/64)) + (15*a*b*tan(x))/(4*((15*a*b)/4 + (3665*a^2)/256 +
(5717*a^3)/(256*b) + (1143*a^4)/(64*b^2) + (235*a^5)/(32*b^3) + (5*a^6)/(4
*b^4))))*(a*b*20i + a^2*8i + b^2*15i)*1i)/(8*b^3) - (atanh((95*a^2*tan(x)*
(- a*b^5 - 5*a^5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/(32
*(2*a*b^4 + (469*a^4*b)/32 + (215*a^5)/32 + (287*a^2*b^3)/32 + (517*a^3*b^
2)/32 + (5*a^6)/(4*b))) + (5*a^3*tan(x)*(- a*b^5 - 5*a^5*b - a^6 - 5*a^2*b
^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/(4*(2*a*b^5 + (215*a^5*b)/32 + (5*a^6
)/4 + (287*a^2*b^4)/32 + (517*a^3*b^3)/32 + (469*a^4*b^2)/32)) + (2*a*tan(
x)*(- a*b^5 - 5*a^5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/
(2*a*b^3 + (517*a^3*b)/32 + (469*a^4)/32 + (287*a^2*b^2)/32 + (215*a^5)...
```

3.18 $\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx$

3.18.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx=-\frac{(2a+3b)x}{2b^2}-\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}b^2}+\frac{\cos(x)\sin(x)}{2b}$$

output
```
-1/2*(2*a+3*b)*x/b^2+1/2*cos(x)*sin(x)/b-(a+b)^(3/2)*arctan(cot(x)*(a+b)^(
1/2)/a^(1/2))/b^2/a^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx=\frac{-4ax-6bx+\frac{4(a+b)^{3/2}\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}+b\sin(2x)}{4b^2}$$

input
```
Integrate[Sin[x]^4/(a + b*Cos[x]^2),x]
```

output
```
(-4*a*x - 6*b*x + (4*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/S
qrt[a] + b*Sin[2*x])/(4*b^2)
```

3.18.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3670, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^4}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{3670}$$
$$-\int \frac{1}{\left(\cot^2(x)+1\right)^2\left((a+b)\cot^2(x)+a\right)}d\cot(x)$$
$$\Big\downarrow \text{316}$$
$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\int \frac{-\left((a+b)\cot^2(x)\right)+a+2b}{\left(\cot^2(x)+1\right)\left((a+b)\cot^2(x)+a\right)}d\cot(x)}{2b}$$
$$\Big\downarrow \text{397}$$
$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{(2a+3b)\int \frac{1}{\cot^2(x)+1}d\cot(x)}{b}}{2b}$$
$$\Big\downarrow \text{216}$$
$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{(2a+3b)\arctan(\cot(x))}{b}}{2b}$$
$$\Big\downarrow \text{218}$$
$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{(2a+3b)\arctan(\cot(x))}{b}}{2b}$$

input
```
Int[Sin[x]^4/(a + b*Cos[x]^2),x]
```

output
```
-1/2*(-(((2*a + 3*b)*ArcTan[Cot[x]])/b) + (2*(a + b)^(3/2)*ArcTan[(Sqrt[a
+ b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*b))/b + Cot[x]/(2*b*(1 + Cot[x]^2))
```

3.18.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.18.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result
default	$\frac{(a+b)^2 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{b^2\sqrt{(a+b)a}} - \frac{-\frac{b\tan(x)}{2\left(\tan^2(x)+1\right)}+\frac{(2a+3b)\arctan(\tan(x))}{2}}{b^2}$
risch	$-\frac{xa}{b^2} - \frac{3x}{2b} - \frac{i e^{2ix}}{8b} + \frac{i e^{-2ix}}{8b} + \frac{\sqrt{-(a+b)a}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)a}-2a-b}{b}\right)}{2b^2} + \frac{\sqrt{-(a+b)a}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)a}-2a-b}{b}\right)}{2ab} -$

input
```
int(sin(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(a+b)^2/b^2/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))-1/b^2*(-1/2*b
*tan(x)/(tan(x)^2+1)+1/2*(2*a+3*b)*arctan(tan(x)))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.52

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{2\,b\cos(x)\sin(x)+(a+b)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\cdots}}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right.}{4\,b^2}\right.$$

input
```
integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*cos(x)*sin(x) + (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b
^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^
2*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 +
a^2)) - 2*(2*a + 3*b)*x)/b^2, 1/2*(b*cos(x)*sin(x) - (a + b)*sqrt((a + b)/
a)*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*sin
(x))) - (2*a + 3*b)*x)/b^2]
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**4/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx = -\frac{(2\,a+3\,b)x}{2\,b^2} + \frac{\tan(x)}{2\,(b\tan(x)^2+b)} + \frac{(a^2+2\,ab+b^2)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^2}$$

input
```
integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*(2*a + 3*b)*x/b^2 + 1/2*tan(x)/(b*tan(x)^2 + b) + (a^2 + 2*a*b + b^2)
*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx = -\frac{(2\,a+3\,b)x}{2\,b^2} + \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{a^2+ab}b^2} + \frac{\tan(x)}{2\,(\tan(x)^2+1)b}$$

input
```
integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sq
rt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*b^2) + 1/2*tan(x)/((t
an(x)^2 + 1)*b)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.10

$$\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx = \frac{\cos(x)\,\sin(x)}{2\,b} - \frac{a\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{b^2} - \frac{3\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{2\,b} - \frac{\mathrm{atanh}\left(\frac{\sin(x)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{\cos(x)\,a^2+2\,\cos(x)\,a\,b+\cos(x)\,b^2}\right)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{a\,b^2}$$

input
```
int(sin(x)^4/(a + b*cos(x)^2),x)
```

output
```
(cos(x)*sin(x))/(2*b) - (a*atan(sin(x)/cos(x)))/b^2 - (3*atan(sin(x)/cos(x
)))/(2*b) - (atanh((sin(x)*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a
^2*cos(x) + b^2*cos(x) + 2*a*b*cos(x)))*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b
^2)^(1/2))/(a*b^2)
```

3.19 $\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx$

3.19.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = -\frac{x}{b}-\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}b}$$

output
```
-x/b-arctan(cot(x)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = \frac{-x+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}}{b}$$

input
```
Integrate[Sin[x]^2/(a + b*Cos[x]^2),x]
```

output
```
(-x + (Sqrt[a + b]*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a])/b
```

3.19.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3670, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3670}$$

$$-\int \frac{1}{\left(\cot^2(x)+1\right)\left((a+b)\cot^2(x)+a\right)}d\cot(x)$$

$$\Big\downarrow \text{ 303}$$

$$\frac{\int \frac{1}{\cot^2(x)+1}d\cot(x)}{b}-\frac{(a+b)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(\cot(x))}{b}-\frac{(a+b)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan(\cot(x))}{b}-\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}b}$$

input
```
Int[Sin[x]^2/(a + b*Cos[x]^2),x]
```

output
```
ArcTan[Cot[x]]/b - (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqr
t[a]*b)
```

3.19.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.19.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{b\sqrt{(a+b)a}}-\frac{\arctan(\tan(x))}{b}$	36
risch	$-\frac{x}{b}-\frac{\sqrt{-(a+b)a}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-(a+b)a}+2a+b}{b}\right)}{2ab}+\frac{\sqrt{-(a+b)a}\,\ln\left(\mathrm{e}^{2ix}-\frac{2i\sqrt{-(a+b)a}-2a-b}{b}\right)}{2ab}$	97

input
```
int(sin(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(a+b)/b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))-1/b*arctan(tan(x)
)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.42

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = \left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2-4\left((2a^2+ab)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-4x}{4b}, -\frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2a+b)\cos(x)^2-a\right)\sqrt{\frac{a+b}{a}}}{2(a+b)\cos(x)\sin(x)}\right)+2x}{2b}\right]$$

input
```
integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*cos(x))*sqrt(-(a + b)/a)*s
in(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*x)/b, -1/2*(sqrt((
a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos
(x)*sin(x))) + 2*x)/b]
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**2/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = \frac{(a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b} - \frac{x}{b}$$

input
```
integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
(a + b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - x/b
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}b} - \frac{x}{b}$$

input
```
integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(a + b)/(
sqrt(a^2 + a*b)*b) - x/b
```

3.19.9 Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.70

$$\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx = -\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\tan(x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\tan(x)}{2\,a^2\,b+2\,a\,b^2}\right)}{b} - \frac{\operatorname{atanh}\left(\frac{2\,a^2\,b\tan(x)\,\sqrt{-a^2-b\,a}}{2\,a^3\,b+2\,a^2\,b^2}\right)\,\sqrt{-a\,(a+b)}}{a\,b}$$

input
```
int(sin(x)^2/(a + b*cos(x)^2),x)
```

output
```
- atan((2*a*b^2*tan(x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(x))/(2*a*b^2 +
2*a^2*b))/b - (atanh((2*a^2*b*tan(x)*(- a*b - a^2)^(1/2))/(2*a^3*b + 2*a^2
*b^2))*(-a*(a + b))^(1/2))/(a*b)
```

3.20 $\int \frac{1}{a+b\cos^2(x)}\,dx$

3.20.1 Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{a+b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
-arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

3.20.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 3660$$

$$-\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \ 218$$

$$-\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

3.20.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.20.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$	21
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}+\frac{\ln\left(e^{2ix}+\frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}$	158

input
```
int(1/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.43

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{4(a^2+ab)}, -\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2
*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)
]
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. 2(29) = 58.

Time = 18.67 (sec) , antiderivative size = 10924, normalized size of antiderivative = 364.13

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-tan(x/2
)/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (-2*tan(x/2)/(b*(tan(x/2)**2 - 1))
, Eq(a, 0)), (a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log
(-sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*
sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a
 + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*
sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt
(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a
+ b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*s
qrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(
a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqr
t(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b
) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-
a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) -
2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b...
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2
+ a*b)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*cos(x)^2),x)
```

output
```
atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

3.21 $\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx$

3.21.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx=-\frac{b\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}-\frac{\cot(x)}{a+b}$$

output
```
-cot(x)/(a+b)-b*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/(a+b)^(3/2)/a^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx=\frac{b\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{3/2}}-\frac{\cot(x)}{a+b}$$

input
```
Integrate[Csc[x]^2/(a + b*Cos[x]^2),x]
```

output
```
(b*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(3/2)) - Cot[x]/
(a + b)
```

3.21.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^2\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \; 3670$$

$$-\int \frac{\cot^2(x)+1}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \; 299$$

$$-\frac{b\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{a+b}-\frac{\cot(x)}{a+b}$$

$$\Big\downarrow \; 218$$

$$-\frac{b\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}-\frac{\cot(x)}{a+b}$$

input
```
Int[Csc[x]^2/(a + b*Cos[x]^2),x]
```

output
```
-((b*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2))) - Cot[
x]/(a + b)
```

3.21.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.21.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{(a+b)\tan(x)}+\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a+b)\sqrt{(a+b)a}}$	39
risch	$-\frac{2i}{(e^{2ix}-1)(a+b)}+\frac{b\ln\left(e^{2ix}-\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)}-\frac{b\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)}$	189

input
```
int(csc(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)/tan(x)+b/(a+b)/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 5.56

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\,b\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)\sin(x)+ \cdots}{4(a^3+2a^2b+ab^2)\sin(x)}, -\frac{\sqrt{a^2+ab}\,b\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\sin(x)+2(a^2+ab)\cos(x)}{2(a^3+2a^2b+ab^2)\sin(x)}\right]$$

input
```
integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 +
 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(
x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) + 4*(a^2 + a*b)*co
s(x))/((a^3 + 2*a^2*b + a*b^2)*sin(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2
*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*sin(x) + 2*(a^2
 + a*b)*cos(x))/((a^3 + 2*a^2*b + a*b^2)*sin(x))]
```

3.21.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx = \int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(csc(x)**2/(a+b*cos(x)**2),x)
```

output
```
Integral(csc(x)**2/(a + b*cos(x)**2), x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx = \frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)} - \frac{1}{(a+b)\tan(x)}$$

input
```
integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
b*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)) - 1/((a + b)*
tan(x))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}(a+b)} - \frac{1}{(a+b)\tan(x)}$$

input
```
integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b/(sqrt(a
^2 + a*b)*(a + b)) - 1/((a + b)*tan(x))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx = \frac{b\,\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}} - \frac{1}{\tan(x)\,(a+b)}$$

input
```
int(1/(sin(x)^2*(a + b*cos(x)^2)),x)
```

output
```
(b*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(1/2)*(a + b)^(3/2)) - 1/(tan(
x)*(a + b))
```

3.22 $\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx$

3.22.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx=-\frac{b^2\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{(a+2b)\cot(x)}{(a+b)^2}-\frac{\cot^3(x)}{3(a+b)}$$

output
```
-(a+2*b)*cot(x)/(a+b)^2-1/3*cot(x)^3/(a+b)-b^2*arctan(cot(x)*(a+b)^(1/2)/a
^(1/2))/(a+b)^(5/2)/a^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{\cot(x)\left(2a+5b+(a+b)\csc^2(x)\right)}{3(a+b)^2}$$

input
```
Integrate[Csc[x]^4/(a + b*Cos[x]^2),x]
```

output
```
(b^2*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) - (Cot[
x]*(2*a + 5*b + (a + b)*Csc[x]^2))/(3*(a + b)^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^4 \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$-\int \frac{\left(\cot^2(x)+1\right)^2}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{300}$$

$$-\int \left(\frac{b^2}{(a+b)^2\left((a+b)\cot^2(x)+a\right)}+\frac{\cot^2(x)}{a+b}+\frac{a+2b}{(a+b)^2}\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{\cot^3(x)}{3(a+b)}-\frac{(a+2b)\cot(x)}{(a+b)^2}$$

input
```
Int[Csc[x]^4/(a + b*Cos[x]^2),x]
```

output
```
-((b^2*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2))) - ((
a + 2*b)*Cot[x])/(a + b)^2 - Cot[x]^3/(3*(a + b))
```

3.22.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.22.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{3(a+b)\tan(x)^3}-\frac{a+2b}{(a+b)^2\tan(x)}+\frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a+b)^2\sqrt{(a+b)a}}$
risch	$-\frac{2i(3b\,e^{4ix}-6a\,e^{2ix}-12b\,e^{2ix}+2a+5b)}{3(e^{2ix}-1)^3(a+b)^2}-\frac{b^2\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)^2}+\frac{b^2\ln\left(e^{2ix}+\frac{-2ia^2-2iab+2a\sqrt{-a^2-}}{b\sqrt{-a^2-}}\right.}{2\sqrt{-a^2-ab}\,(a+b)}$

input
```
int(csc(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a+b)/tan(x)^3-(a+2*b)/(a+b)^2/tan(x)+b^2/(a+b)^2/((a+b)*a)^(1/2)*arc
tan(a*tan(x)/((a+b)*a)^(1/2))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 6.49

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{4\,(2\,a^3+7\,a^2b+5\,ab^2)\cos(x)^3+3\,(b^2\cos(x)^2-b^2)\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2}{b^2\cos(x)^4+}\right.}{12\,(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^4+3\,a^3b+3\,a^2}\right.$$

input
```
integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/12*(4*(2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^3 + 3*(b^2*cos(x)^2 - b^2)*sqr
t(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(
x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b
^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) - 12*(a^3 + 3*a^2*b + 2*a*b^2)
*cos(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*cos(x)^2)*sin(x)), 1/6*(2*(2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^3 +
3*(b^2*cos(x)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)
/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*sin(x) - 6*(a^3 + 3*a^2*b + 2*a*b^2)*cos
(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*cos(x)^2)*sin(x))]
```

3.22.6 Sympy [F]

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx = \int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(csc(x)**4/(a+b*cos(x)**2),x)
```

output
```
Integral(csc(x)**4/(a + b*cos(x)**2), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx = \frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+2\,ab+b^2)} - \frac{3\,(a+2\,b)\tan{(x)}^2+a+b}{3\,(a^2+2\,ab+b^2)\tan{(x)}^3}$$

input
```
integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
b^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2))
 - 1/3*(3*(a + 2*b)*tan(x)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(x)^3)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.48

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{(a^2+2\,ab+b^2)\sqrt{a^2+ab}} - \frac{3\,a\tan{(x)}^2+6\,b\tan{(x)}^2+a+b}{3\,(a^2+2\,ab+b^2)\tan{(x)}^3}$$

input
```
integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^2/((a^2
 + 2*a*b + b^2)*sqrt(a^2 + a*b)) - 1/3*(3*a*tan(x)^2 + 6*b*tan(x)^2 + a +
b)/((a^2 + 2*a*b + b^2)*tan(x)^3)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx = \frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)\,(a^2+2\,a\,b+b^2)}{(a+b)^{5/2}}\right)}{\sqrt{a}\,(a+b)^{5/2}} - \frac{\frac{1}{3\,(a+b)}+\frac{\tan(x)^2\,(a+2\,b)}{(a+b)^2}}{\tan{(x)}^3}$$

input
```
int(1/(sin(x)^4*(a + b*cos(x)^2)),x)
```

output
```
(b^2*atan((a^(1/2)*tan(x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(a^(1/2)*(a
 + b)^(5/2)) - (1/(3*(a + b)) + (tan(x)^2*(a + 2*b))/(a + b)^2)/tan(x)^3
```

3.22. $\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx$

3.23 $\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx$

3.23.1 Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = -\frac{b^3\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}} - \frac{(a^2+3ab+3b^2)\cot(x)}{(a+b)^3} - \frac{(2a+3b)\cot^3(x)}{3(a+b)^2} - \frac{\cot^5(x)}{5(a+b)}$$

output
```
-(a^2+3*a*b+3*b^2)*cot(x)/(a+b)^3-1/3*(2*a+3*b)*cot(x)^3/(a+b)^2-1/5*cot(x
)^5/(a+b)-b^3*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/(a+b)^(7/2)/a^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = \frac{b^3\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{7/2}} - \frac{\cot(x)\left(8a^2+26ab+33b^2+(4a^2+13ab+9b^2)\csc^2(x)+3(a+b)^2\csc^4(x)\right)}{15(a+b)^3}$$

input
```
Integrate[Csc[x]^6/(a + b*Cos[x]^2),x]
```

output
```
(b^3*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(7/2)) - (Cot[
x]*(8*a^2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Csc[x]^2 + 3*(a + b
)^2*Csc[x]^4))/(15*(a + b)^3)
```

3.23.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x+\frac{\pi}{2}\right)^6\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$-\int \frac{\left(\cot^2(x)+1\right)^3}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{300}$$

$$-\int \left(\frac{\cot^4(x)}{a+b}+\frac{(2a+3b)\cot^2(x)}{(a+b)^2}+\frac{a^2+3ba+3b^2}{(a+b)^3}+\frac{b^3}{(a+b)^3\left((a+b)\cot^2(x)+a\right)}\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a^2+3ab+3b^2\right)\cot(x)}{(a+b)^3}-\frac{b^3\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}}-\frac{\cot^5(x)}{5(a+b)}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}$$

input
```
Int[Csc[x]^6/(a + b*Cos[x]^2),x]
```

output
```
-((b^3*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2))) - ((
a^2 + 3*a*b + 3*b^2)*Cot[x])/(a + b)^3 - ((2*a + 3*b)*Cot[x]^3)/(3*(a + b)
^2) - Cot[x]^5/(5*(a + b))
```

3.23.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.23.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{5(a+b)\tan(x)^5}-\frac{2a+3b}{3(a+b)^2\tan(x)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(x)}+\frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a+b)^3\sqrt{(a+b)a}}$
risch	$-\frac{2i(15b^2e^{8ix}-30ab\,e^{6ix}-90b^2e^{6ix}+80a^2e^{4ix}+230\,e^{4ix}ab+240b^2e^{4ix}-40\,e^{2ix}a^2-130b\,e^{2ix}a-150b^2e^{2ix}+8a^2+26ab+33b^2)}{15(a+b)^3(e^{2ix}-1)^5}+\frac{b^3\ \text{l}}{}$

input
```
int(csc(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/(a+b)/tan(x)^5-1/3*(2*a+3*b)/(a+b)^2/tan(x)^3-(a^2+3*a*b+3*b^2)/(a+b)
^3/tan(x)+b^3/(a+b)^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(77) = 154$.

Time = 0.30 (sec) , antiderivative size = 610, normalized size of antiderivative = 6.85

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[-\frac{4\,(8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3)\cos(x)^5-20\,(4\,a^4+17\,a^3b+28\,a^2b^2+15\,ab^3)\cos(x)^3+15\,(b^3\cos}{60\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4}\right.$$
$$-\frac{2\,(8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3)\cos(x)^5-10\,(4\,a^4+17\,a^3b+28\,a^2b^2+15\,ab^3)\cos(x)^3+15\,(b^3\cos}{30\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2}$$

input
```
integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^5 - 20*(4*a^4
+ 17*a^3*b + 28*a^2*b^2 + 15*a*b^3)*cos(x)^3 + 15*(b^3*cos(x)^4 - 2*b^3*co
s(x)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*
a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)
*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) + 60*(a^4 + 4
*a^3*b + 6*a^2*b^2 + 3*a*b^3)*cos(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*
b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^4 - 2
*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)*sin(x)), -1/30*
(2*(8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^5 - 10*(4*a^4 + 17*a^
3*b + 28*a^2*b^2 + 15*a*b^3)*cos(x)^3 + 15*(b^3*cos(x)^4 - 2*b^3*cos(x)^2
+ b^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b
)*cos(x)*sin(x)))*sin(x) + 30*(a^4 + 4*a^3*b + 6*a^2*b^2 + 3*a*b^3)*cos(x)
)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3
*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^4 - 2*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*
b^3 + a*b^4)*cos(x)^2)*sin(x))]
```

3.23.6 Sympy [F]

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = \int \frac{\csc^6{(x)}}{a+b\cos^2{(x)}}\,dx$$

input
```
integrate(csc(x)**6/(a+b*cos(x)**2),x)
```

output
```
Integral(csc(x)**6/(a + b*cos(x)**2), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = \frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{(a+b)a}} - \frac{15\,(a^2+3ab+3b^2)\tan{(x)}^4+5\,(2a^2+5ab+3b^2)\tan{(x)}^2+3a^2+6ab+3b^2}{15\,(a^3+3a^2b+3ab^2+b^3)\tan{(x)}^5}$$

input
```
integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
b^3*arctan(a*tan(x)/sqrt((a + b)*a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt
((a + b)*a)) - 1/15*(15*(a^2 + 3*a*b + 3*b^2)*tan(x)^4 + 5*(2*a^2 + 5*a*b
+ 3*b^2)*tan(x)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*tan(x)^5)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(77) = 154.

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.75

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{(a^3+3a^2b+3ab^2+b^3)\sqrt{a^2+ab}} - \frac{15a^2\tan{(x)}^4+45ab\tan{(x)}^4+45b^2\tan{(x)}^4+10a^2\tan{(x)}^2+25ab\tan{(x)}^2+15b^2\tan{(x)}^2+3a^2+}{15\,(a^3+3a^2b+3ab^2+b^3)\tan{(x)}^5}$$

input
```
integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^3/((a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) - 1/15*(15*a^2*tan(x)^4 + 45*
a*b*tan(x)^4 + 45*b^2*tan(x)^4 + 10*a^2*tan(x)^2 + 25*a*b*tan(x)^2 + 15*b^
2*tan(x)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(x
)^5)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)}{\sqrt{a}\,(a+b)^{7/2}} - \frac{\frac{1}{5\,(a+b)}+\frac{\tan(x)^2\,(2\,a+3\,b)}{3\,(a+b)^2}+\frac{\tan(x)^4\,\left(a^2+3\,a\,b+3\,b^2\right)}{(a+b)^3}}{\tan(x)^5}$$

input
```
int(1/(sin(x)^6*(a + b*cos(x)^2)),x)
```

output
```
(b^3*atan((a^(1/2)*tan(x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))
/(a^(1/2)*(a + b)^(7/2)) - (1/(5*(a + b)) + (tan(x)^2*(2*a + 3*b))/(3*(a +
 b)^2) + (tan(x)^4*(3*a*b + a^2 + 3*b^2))/(a + b)^3)/tan(x)^5
```

3.24 $\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx$

3.24.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = -\frac{\arctan\left(\frac{1+\sqrt[3]{6}\cos(x)}{\sqrt{3}}\right)}{2\sqrt[3]{2}3^{5/6}}+\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cos(x)+3^{2/3}\cos^2(x)\right)}{12\sqrt[3]{6}}$$

output
```
-1/12*arctan(1/3*(1+6^(1/3)*cos(x))*3^(1/2))*2^(2/3)*3^(1/6)+1/36*ln(2^(2/
3)-3^(1/3)*cos(x))*6^(2/3)-1/72*ln(2*2^(1/3)+2^(2/3)*3^(1/3)*cos(x)+3^(2/3
)*cos(x)^2)*6^(2/3)
```

3.24.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = \frac{1}{72}\left(-62^{2/3}\sqrt[6]{3}\arctan\left(\frac{1+\sqrt[3]{6}\cos(x)}{\sqrt{3}}\right) + 6^{2/3}\left(2\log\left(2-\sqrt[3]{6}\cos(x)\right)-\log\left(4+2\sqrt[3]{6}\cos(x)+6^{2/3}\cos^2(x)\right)\right)\right)$$

input
```
Integrate[Sin[x]/(4 - 3*Cos[x]^3),x]
```

output
```
(-6*2^(2/3)*3^(1/6)*ArcTan[(1 + 6^(1/3)*Cos[x])/Sqrt[3]] + 6^(2/3)*(2*Log[
2 - 6^(1/3)*Cos[x]] - Log[4 + 2*6^(1/3)*Cos[x] + 6^(2/3)*Cos[x]^2]))/72
```

3.24.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 25, 3702, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)}{4-3\sin\left(x+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)}{4-3\sin\left(x+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 3702}$$

$$-\int \frac{1}{4-3\cos^3(x)}d\cos(x)$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{\int \frac{\sqrt[3]{3}\cos(x)+2\ 2^{2/3}}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{6\sqrt[3]{2}}-\frac{\int \frac{1}{2^{2/3}-\sqrt[3]{3}\cos(x)}d\cos(x)}{6\sqrt[3]{2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\int \frac{\sqrt[3]{3}\cos(x)+2\ 2^{2/3}}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{6\sqrt[3]{2}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\frac{3\int \frac{1}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{\sqrt[3]{2}}+\frac{\int \frac{2^{2/3}\sqrt[3]{3}\left(\sqrt[3]{6}\cos(x)+1\right)}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{2\sqrt[3]{3}}}{6\sqrt[3]{2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\frac{3\int \frac{1}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{6}\cos(x)+1}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{\sqrt[3]{2}}}{6\sqrt[3]{2}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\frac{\int \frac{\sqrt[3]{6}\cos(x)+1}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{\sqrt[3]{2}}-3^{2/3}\int \frac{1}{-\left(\sqrt[3]{6}\cos(x)+1\right)^2-3}d\left(\sqrt[3]{6}\cos(x)+1\right)}{6\sqrt[3]{2}}$$

$$\Big\downarrow \; 217$$

$$\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\frac{\int \frac{\sqrt[3]{6}\cos(x)+1}{3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}}d\cos(x)}{\sqrt[3]{2}}+\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{6}\cos(x)+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{6}\cos(x)+1}{\sqrt{3}}\right)+\frac{\log\left(3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}\right)}{2\sqrt[3]{3}}}{6\sqrt[3]{2}}$$

input
```
Int[Sin[x]/(4 - 3*Cos[x]^3),x]
```

output
```
Log[2^(2/3) - 3^(1/3)*Cos[x]]/(6*6^(1/3)) - (3^(1/6)*ArcTan[(1 + 6^(1/3)*C
os[x])/Sqrt[3]] + Log[2*2^(1/3) + 2^(2/3)*3^(1/3)*Cos[x] + 3^(2/3)*Cos[x]^
2]/(2*3^(1/3)))/(6*2^(1/3))
```

3.24. $\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx$

3.24.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.24.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{i\left(\sum_{\_R=\text{RootOf}\left(162\_Z^3+i\right)}\_R\ln\left(\mathrm{e}^{2ix}+12i\_R\,\mathrm{e}^{ix}+1\right)\right)}{2}$
derivativedivides	$\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos^2(x)+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cos(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}-\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cos(x)}{2}+1\right)}{3}\right)}{12}$
default	$\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos^2(x)+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cos(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}-\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cos(x)}{2}+1\right)}{3}\right)}{12}$

input
```
int(sin(x)/(4-3*cos(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*sum(_R*ln(exp(2*I*x)+12*I*_R*exp(I*x)+1),_R=RootOf(162*_Z^3+I))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = -\frac{1}{12}\cdot 6^{\frac{1}{6}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 6^{\frac{1}{6}}\left(6^{\frac{2}{3}}\sqrt{2}\cos(x)+6^{\frac{1}{3}}\sqrt{2}\right)\right)-\frac{1}{72}\cdot 6^{\frac{2}{3}}\log\left(-3\cos(x)^2-6^{\frac{2}{3}}\cos(x)-2\cdot 6^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 6^{\frac{2}{3}}\log\left(6^{\frac{2}{3}}-3\cos(x)\right)$$

input
```
integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="fricas")
```

output
```
-1/12*6^(1/6)*sqrt(2)*arctan(1/6*6^(1/6)*(6^(2/3)*sqrt(2)*cos(x) + 6^(1/3)
*sqrt(2))) - 1/72*6^(2/3)*log(-3*cos(x)^2 - 6^(2/3)*cos(x) - 2*6^(1/3)) +
1/36*6^(2/3)*log(6^(2/3) - 3*cos(x))
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = \frac{6^{\frac{2}{3}}\log\left(\cos\left(x\right)-\frac{6^{\frac{2}{3}}}{3}\right)}{36} - \frac{6^{\frac{2}{3}}\log\left(36\cos^2\left(x\right)+12\cdot 6^{\frac{2}{3}}\cos\left(x\right)+24\cdot\sqrt[3]{6}\right)}{72} - \frac{2^{\frac{2}{3}}\cdot\sqrt[6]{3}\operatorname{atan}\left(\frac{\sqrt[3]{2}\cdot 3^{\frac{5}{6}}\cos\left(x\right)}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(sin(x)/(4-3*cos(x)**3),x)
```

output
```
6**(2/3)*log(cos(x) - 6**(2/3)/3)/36 - 6**(2/3)*log(36*cos(x)**2 + 12*6**(
2/3)*cos(x) + 24*6**(1/3))/72 - 2**(2/3)*3**(1/6)*atan(2**(1/3)*3**(5/6)*c
os(x)/3 + sqrt(3)/3)/12
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = -\frac{1}{72}\cdot 4^{\frac{1}{3}}3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}\cos\left(x\right)^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\cos\left(x\right)+4^{\frac{2}{3}}\right) + \frac{1}{36}\cdot 4^{\frac{1}{3}}3^{\frac{2}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\cos\left(x\right)-4^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}3^{\frac{1}{6}}\left(2\cdot 3^{\frac{2}{3}}\cos\left(x\right)+4^{\frac{1}{3}}3^{\frac{1}{3}}\right)\right)$$

input
```
integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="maxima")
```

output
```
-1/72*4^(1/3)*3^(2/3)*log(3^(2/3)*cos(x)^2 + 4^(1/3)*3^(1/3)*cos(x) + 4^(2
/3)) + 1/36*4^(1/3)*3^(2/3)*log(1/3*3^(2/3)*(3^(1/3)*cos(x) - 4^(1/3))) -
1/12*4^(1/3)*3^(1/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*3^(2/3)*cos(x) + 4^(1/
3)*3^(1/3)))
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = -\frac{1}{12}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{4}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{2}{3}}\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}+2\cos\left(x\right)\right)\right) - \frac{1}{72}\cdot 36^{\frac{1}{3}}\log\left(\cos\left(x\right)^{2}+\left(\frac{4}{3}\right)^{\frac{1}{3}}\cos\left(x\right)+\left(\frac{4}{3}\right)^{\frac{2}{3}}\right) + \frac{1}{12}\left(\frac{4}{3}\right)^{\frac{1}{3}}\log\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}-\cos\left(x\right)\right)$$

input
```
integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*(4/3)^(1/3)*arctan(1/4*sqrt(3)*(4/3)^(2/3)*((4/3)^(1/3) + 2*
cos(x))) - 1/72*36^(1/3)*log(cos(x)^2 + (4/3)^(1/3)*cos(x) + (4/3)^(2/3))
+ 1/12*(4/3)^(1/3)*log((4/3)^(1/3) - cos(x))
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = \frac{6^{2/3}\ln\left(\cos\left(x\right)-\frac{6^{2/3}}{3}\right)}{36} + \frac{6^{2/3}\ln\left(\cos\left(x\right)-\frac{6^{2/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{72} - \frac{6^{2/3}\ln\left(\cos\left(x\right)+\frac{6^{2/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{72}$$

input
```
int(-sin(x)/(3*cos(x)^3 - 4),x)
```

output
```
(6^(2/3)*log(cos(x) - 6^(2/3)/3))/36 + (6^(2/3)*log(cos(x) - (6^(2/3)*(3^(
1/2)*1i - 1))/6)*(3^(1/2)*1i - 1))/72 - (6^(2/3)*log(cos(x) + (6^(2/3)*(3^
(1/2)*1i + 1))/6)*(3^(1/2)*1i + 1))/72
```

3.24. $\int \frac{\sin(x)}{4-3\cos^3(x)}\, dx$

3.25 $\int \frac{1}{1-\cos^2(x)} dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 4

$$\int \frac{1}{1-\cos^2(x)} dx = -\cot(x)$$

output
```
-cot(x)
```

3.25.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cos^2(x)} dx = -\cot(x)$$

input
```
Integrate[(1 - Cos[x]^2)^(-1),x]
```

output
```
-Cot[x]
```

3.25.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\int \csc^2(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \csc(x)^2dx$$
$$\Big\downarrow \text{ 4254}$$
$$-\int 1d\cot(x)$$
$$\Big\downarrow \text{ 24}$$
$$-\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(-1),x]
```

output
```
-Cot[x]
```

3.25.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
parallelrisch	$-\cot\left(x\right)$	5
default	$-\frac{1}{\tan\left(x\right)}$	7
risch	$-\frac{2i}{{\mathrm e}^{2ix}-1}$	13
norman	$\frac{-\frac{1}{2}+\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)}$	18

input
```
int(1/(1-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{\cos(x)}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)^2),x, algorithm="fricas")
```

output
```
-cos(x)/sin(x)
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(3) = 6.

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{1}{1-\cos^2(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**2),x)
```

output
```
tan(x/2)/2 - 1/(2*tan(x/2))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{1}{\tan(x)}$$

input
```
integrate(1/(1-cos(x)^2),x, algorithm="maxima")
```

output
```
-1/tan(x)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{1}{\tan(x)}$$

input
```
integrate(1/(1-cos(x)^2),x, algorithm="giac")
```

output
```
-1/tan(x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\cot(x)$$

input
```
int(-1/(cos(x)^2 - 1),x)
```

output
```
-cot(x)
```

3.26 $\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx = -\cot(x) - \frac{\cot^3(x)}{3}$$

output
```
-cot(x)-1/3*cot(x)^3
```

3.26.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx = -\frac{2\cot(x)}{3} - \frac{1}{3}\cot(x)\csc^2(x)$$

input
```
Integrate[(1 - Cos[x]^2)^(-2),x]
```

output
```
(-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

3.26.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\int \csc^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^4dx$$

$$\Big\downarrow \text{4254}$$

$$-\int \left(\cot^2(x)+1\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(-2),x]
```

output
```
-Cot[x] - Cot[x]^3/3
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{\tan(x)}-\frac{1}{3\tan(x)^3}$	14
parallelrisch	$\frac{2(\cot^3(x))}{3}-\cot(x)(\csc^2(x))$	16
risch	$\frac{4i(3\,e^{2ix}-1)}{3(e^{2ix}-1)^3}$	22
norman	$\frac{-\frac{1}{24}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34

input
```
int(1/(1-cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/tan(x)-1/3/tan(x)^3
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2}\,dx = -\frac{2\,\cos\left(x\right)^3 - 3\,\cos\left(x\right)}{3\,\left(\cos\left(x\right)^2 - 1\right)\,\sin\left(x\right)}$$

input
```
integrate(1/(1-cos(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2}\,dx = \frac{\tan^3\left(\frac{x}{2}\right)}{24} + \frac{3\tan\left(\frac{x}{2}\right)}{8} - \frac{3}{8\tan\left(\frac{x}{2}\right)} - \frac{1}{24\tan^3\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**2)**2,x)
```

output
```
tan(x/2)**3/24 + 3*tan(x/2)/8 - 3/(8*tan(x/2)) - 1/(24*tan(x/2)**3)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2}\,dx = -\frac{3\,\tan\left(x\right)^2 + 1}{3\,\tan\left(x\right)^3}$$

input
```
integrate(1/(1-cos(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2}\,dx = -\frac{3\,\tan\left(x\right)^2+1}{3\,\tan\left(x\right)^3}$$

input
```
integrate(1/(1-cos(x)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

3.26.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(1-\cos^2(x)\right)^2}\,dx = -\frac{\cot(x)\,\left(\cot(x)^2+3\right)}{3}$$

input
```
int(1/(cos(x)^2 - 1)^2,x)
```

output
```
-(cot(x)*(cot(x)^2 + 3))/3
```

3.27 $\int \frac{1}{(1-\cos^2(x))^3}\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{(1-\cos^2(x))^3}\,dx = -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}$$

output
```
-cot(x)-2/3*cot(x)^3-1/5*cot(x)^5
```

3.27.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(1-\cos^2(x))^3}\,dx = -\frac{8\cot(x)}{15} - \frac{4}{15}\cot(x)\csc^2(x) - \frac{1}{5}\cot(x)\csc^4(x)$$

input
```
Integrate[(1 - Cos[x]^2)^(-3),x]
```

output
```
(-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

3.27.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\cos^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^3}dx$$

$$\Big\downarrow \text{3654}$$

$$\int \csc^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^6dx$$

$$\Big\downarrow \text{4254}$$

$$-\int \left(\cot^4(x)+2\cot^2(x)+1\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(-3),x]
```

output
```
-Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{\tan(x)}-\frac{1}{5\tan(x)^{5}}-\frac{2}{3\tan(x)^{3}}$	20
parallelrisch	$-\frac{\cot(x)\left(\csc^{4}(x)\right)(8+\cos(4x)-6\cos(2x))}{15}$	21
risch	$-\frac{16i\left(10\,\mathrm{e}^{4ix}-5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}-1\right)^{5}}$	29
norman	$\frac{-\frac{1}{160}-\frac{5\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{96}-\frac{5\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{96}+\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{160}}{\tan\left(\frac{x}{2}\right)^{5}}$	50

input
```
int(1/(1-cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/tan(x)-1/5/tan(x)^5-2/3/tan(x)^3
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{(1-\cos^2(x))^3} \, dx = -\frac{8 \cos(x)^5 - 20 \cos(x)^3 + 15 \cos(x)}{15 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \sin(x)}$$

input
```
integrate(1/(1-cos(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*
sin(x))
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(19) = 38$.

Time = 1.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \frac{1}{(1-\cos^2(x))^3} \, dx = \frac{\tan^5\left(\frac{x}{2}\right)}{160} + \frac{5\tan^3\left(\frac{x}{2}\right)}{96} + \frac{5\tan\left(\frac{x}{2}\right)}{16} - \frac{5}{16\tan\left(\frac{x}{2}\right)} - \frac{5}{96\tan^3\left(\frac{x}{2}\right)} - \frac{1}{160\tan^5\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**2)**3,x)
```

output
```
tan(x/2)**5/160 + 5*tan(x/2)**3/96 + 5*tan(x/2)/16 - 5/(16*tan(x/2)) - 5/(
96*tan(x/2)**3) - 1/(160*tan(x/2)**5)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-\cos^2(x))^3} \, dx = -\frac{15 \tan(x)^4 + 10 \tan(x)^2 + 3}{15 \tan(x)^5}$$

input
```
integrate(1/(1-cos(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-\cos^2(x))^3}\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

input
```
integrate(1/(1-cos(x)^2)^3,x, algorithm="giac")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

3.27.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1-\cos^2(x))^3}\,dx = -\frac{\cot(x)^5}{5} - \frac{2\cot(x)^3}{3} - \cot(x)$$

input
```
int(-1/(cos(x)^2 - 1)^3,x)
```

output
```
- cot(x) - (2*cot(x)^3)/3 - cot(x)^5/5
```

3.28 $\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx$

3.28.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = -\frac{a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}} + \frac{(a^2-ab+b^2)\sin(x)}{b^3} + \frac{(a-2b)\sin^3(x)}{3b^2} + \frac{\sin^5(x)}{5b}$$

output
```
(a^2-a*b+b^2)*sin(x)/b^3+1/3*(a-2*b)*sin(x)^3/b^2+1/5*sin(x)^5/b-a^3*arcta
nh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = \frac{a^3\left(\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)-\log\left(\sqrt{a+b}+\sqrt{b}\sin(x)\right)\right)}{2b^{7/2}\sqrt{a+b}} + \frac{(8a^2-6ab+5b^2)\sin(x)}{8b^3} + \frac{(-4a+5b)\sin(3x)}{48b^2} + \frac{\sin(5x)}{80b}$$

input
```
Integrate[Cos[x]^7/(a + b*Cos[x]^2),x]
```

output
```
(a^3*(Log[Sqrt[a + b] - Sqrt[b]*Sin[x]] - Log[Sqrt[a + b] + Sqrt[b]*Sin[x]
]))/(2*b^(7/2)*Sqrt[a + b]) + ((8*a^2 - 6*a*b + 5*b^2)*Sin[x])/(8*b^3) + (
(-4*a + 5*b)*Sin[3*x])/(48*b^2) + Sin[5*x]/(80*b)
```

3.28.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^7}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\int \frac{\left(1-\sin^2(x)\right)^3}{a-b\sin^2(x)+b}d\sin(x)$$

$$\Big\downarrow \text{ 300}$$

$$\int \left(-\frac{a^3}{b^3\left(a-b\sin^2(x)+b\right)}+\frac{a^2-ab+b^2}{b^3}+\frac{(a-2b)\sin^2(x)}{b^2}+\frac{\sin^4(x)}{b}\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\sin(x)}{b^3}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}$$

input
```
Int[Cos[x]^7/(a + b*Cos[x]^2),x]
```

output
```
-((a^3*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b])) + ((a
^2 - a*b + b^2)*Sin[x])/b^3 + ((a - 2*b)*Sin[x]^3)/(3*b^2) + Sin[x]^5/(5*b
)
```

3.28.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.28.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
default	$\frac{\frac{\left(\sin^{5}\left(x\right)\right) b^{2}}{5}+\frac{a b \left(\sin^{3}\left(x\right)\right)}{3}-\frac{2 b^{2} \left(\sin^{3}\left(x\right)\right)}{3}+a^{2} \sin \left(x\right)-b \sin \left(x\right) a+b^{2} \sin \left(x\right)}{b^{3}}-\frac{a^{3} \operatorname{arctanh}\left(\frac{b \sin \left(x\right)}{\sqrt{\left(a+b\right) b}}\right)}{b^{3} \sqrt{\left(a+b\right) b}}$
risch	$-\frac{i {\mathrm e}^{i x} a^{2}}{2 b^{3}}+\frac{3 i {\mathrm e}^{i x} a}{8 b^{2}}-\frac{5 i {\mathrm e}^{i x}}{16 b}+\frac{i {\mathrm e}^{-i x} a^{2}}{2 b^{3}}-\frac{3 i {\mathrm e}^{-i x} a}{8 b^{2}}+\frac{5 i {\mathrm e}^{-i x}}{16 b}+\frac{a^{3} \ln \left({\mathrm e}^{2 i x}-\frac{2 i \left(a+b\right) {\mathrm e}^{i x}}{\sqrt{a b+b^{2}}}-1\right)}{2 \sqrt{a b+b^{2}}\, b^{3}}-\frac{a^{3} \ln \left({\mathrm e}^{2 i x}+\frac{2 i \left(a+b\right) {\mathrm e}^{i x}}{\sqrt{a b+b^{2}}}-1\right)}{2 \sqrt{a b+b^{2}}\, b^{3}}$

input
```
int(cos(x)^7/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*sin(x)^5*b^2+1/3*a*b*sin(x)^3-2/3*b^2*sin(x)^3+a^2*sin(x)-b*sin
(x)*a+b^2*sin(x))-a^3/b^3/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2)
)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.32

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{15\sqrt{ab+b^2}a^3\log\left(-\frac{b\cos(x)^2+2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)+2\left(3\left(ab^3+b^4\right)\cos(x)^4+15a^3b+5a^2b^2-2ab^3+8b\right.}{30\left(ab^4+b^5\right)}\right.$$

input
```
integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(a*b + b^2)*a^3*log(-(b*cos(x)^2 + 2*sqrt(a*b + b^2)*sin(x)
- a - 2*b)/(b*cos(x)^2 + a)) + 2*(3*(a*b^3 + b^4)*cos(x)^4 + 15*a^3*b + 5*
a^2*b^2 - 2*a*b^3 + 8*b^4 - (5*a^2*b^2 + a*b^3 - 4*b^4)*cos(x)^2)*sin(x))/
(a*b^4 + b^5), 1/15*(15*sqrt(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*sin(x
)/(a + b)) + (3*(a*b^3 + b^4)*cos(x)^4 + 15*a^3*b + 5*a^2*b^2 - 2*a*b^3 +
8*b^4 - (5*a^2*b^2 + a*b^3 - 4*b^4)*cos(x)^2)*sin(x))/(a*b^4 + b^5)]
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**7/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = \frac{a^3\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}b^3} + \frac{3b^2\sin(x)^5+5\left(ab-2b^2\right)\sin(x)^3+15\left(a^2-ab+b^2\right)\sin(x)}{15b^3}$$

input
```
integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*a^3*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sq
rt((a + b)*b)*b^3) + 1/15*(3*b^2*sin(x)^5 + 5*(a*b - 2*b^2)*sin(x)^3 + 15*
(a^2 - a*b + b^2)*sin(x))/b^3
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = \frac{a^3\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^3} + \frac{3\,b^4\sin{(x)}^5+5\,ab^3\sin{(x)}^3-10\,b^4\sin{(x)}^3+15\,a^2b^2\sin{(x)}-15\,ab^3\sin{(x)}+15\,b^4\sin{(x)}}{15\,b^5}$$

input
```
integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
a^3*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^3) + 1/15*(3*b^4
*sin(x)^5 + 5*a*b^3*sin(x)^3 - 10*b^4*sin(x)^3 + 15*a^2*b^2*sin(x) - 15*a*
b^3*sin(x) + 15*b^4*sin(x))/b^5
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = \frac{\sin{(x)}^5}{5\,b} + \sin{(x)}^3\left(\frac{a+b}{3\,b^2}-\frac{1}{b}\right) + \sin{(x)}\left(\frac{3}{b}+\frac{(a+b)\,\left(\frac{a+b}{b^2}-\frac{3}{b}\right)}{b}\right) + \frac{a^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,1\mathrm{i}}{\sqrt{a+b}}\right)\,1\mathrm{i}}{b^{7/2}\,\sqrt{a+b}}$$

input
```
int(cos(x)^7/(a + b*cos(x)^2),x)
```

output
```
sin(x)^5/(5*b) + sin(x)^3*((a + b)/(3*b^2) - 1/b) + sin(x)*(3/b + ((a + b)
*((a + b)/b^2 - 3/b))/b) + (a^3*atan((b^(1/2)*sin(x)*1i)/(a + b)^(1/2))*1i
)/(b^(7/2)*(a + b)^(1/2))
```

3.29 $\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx$

3.29.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx=\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

output
```
-(a-b)*sin(x)/b^2-1/3*sin(x)^3/b+a^2*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b
^(5/2)/(a+b)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.54

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx=\frac{\frac{6a^2\left(-\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)+\log\left(\sqrt{a+b}+\sqrt{b}\sin(x)\right)\right)}{\sqrt{a+b}}+3\sqrt{b}(-4a+3b)\sin(x)+b^{3/2}\sin(3x)}{12b^{5/2}}$$

input
```
Integrate[Cos[x]^5/(a + b*Cos[x]^2),x]
```

output
```
((6*a^2*(-Log[Sqrt[a + b] - Sqrt[b]*Sin[x]] + Log[Sqrt[a + b] + Sqrt[b]*Si
n[x]]))/Sqrt[a + b] + 3*Sqrt[b]*(-4*a + 3*b)*Sin[x] + b^(3/2)*Sin[3*x])/(1
2*b^(5/2))
```

3.29.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^5}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{\left(1-\sin^2(x)\right)^2}{a-b\sin^2(x)+b}d\sin(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{a^2}{b^2\left(a-b\sin^2(x)+b\right)}-\frac{a-b}{b^2}-\frac{\sin^2(x)}{b}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

input
```
Int[Cos[x]^5/(a + b*Cos[x]^2),x]
```

output
```
(a^2*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) - ((a -
b)*Sin[x])/b^2 - Sin[x]^3/(3*b)
```

3.29.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.29.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\frac{b\left(\sin^{3}(x)\right)}{3}+\sin(x)a-\sin(x)b}{b^{2}}+\frac{a^{2}\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)b}}\right)}{b^{2}\sqrt{(a+b)b}}$	50
risch	$\frac{ie^{ix}a}{2b^{2}}-\frac{3ie^{ix}}{8b}-\frac{ie^{-ix}a}{2b^{2}}+\frac{3ie^{-ix}}{8b}+\frac{a^{2}\ln\left(e^{2ix}+\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^{2}}}-1\right)}{2\sqrt{ab+b^{2}}\,b^{2}}-\frac{a^{2}\ln\left(e^{2ix}-\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^{2}}}-1\right)}{2\sqrt{ab+b^{2}}\,b^{2}}+\frac{\sin(3x)}{12b}$	147

input
```
int(cos(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(1/3*b*sin(x)^3+sin(x)*a-sin(x)*b)+a^2/b^2/((a+b)*b)^(1/2)*arctanh(
b*sin(x)/((a+b)*b)^(1/2))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.41

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{3\sqrt{ab+b^2}a^2\log\left(-\frac{b\cos(x)^2-2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)-2\left(3a^2b+ab^2-2b^3-(ab^2+b^3)\cos(x)^2\right)\sin(x)}{6(ab^3+b^4)},\right.$$
$$\left.-\frac{3\sqrt{-ab-b^2}a^2\arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right)+\left(3a^2b+ab^2-2b^3-(ab^2+b^3)\cos(x)^2\right)\sin(x)}{3(ab^3+b^4)}\right]$$

input
```
integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a*b + b^2)*a^2*log(-(b*cos(x)^2 - 2*sqrt(a*b + b^2)*sin(x) -
a - 2*b)/(b*cos(x)^2 + a)) - 2*(3*a^2*b + a*b^2 - 2*b^3 - (a*b^2 + b^3)*co
s(x)^2)*sin(x))/(a*b^3 + b^4), -1/3*(3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a
*b - b^2)*sin(x)/(a + b)) + (3*a^2*b + a*b^2 - 2*b^3 - (a*b^2 + b^3)*cos(x
)^2)*sin(x))/(a*b^3 + b^4)]
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**5/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx = -\frac{a^2\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}b^2} - \frac{b\sin(x)^3+3(a-b)\sin(x)}{3b^2}$$

input
```
integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*a^2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(s
qrt((a + b)*b)*b^2) - 1/3*(b*sin(x)^3 + 3*(a - b)*sin(x))/b^2
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx = -\frac{a^2\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^2} - \frac{b^2\sin(x)^3+3ab\sin(x)-3b^2\sin(x)}{3b^3}$$

input
```
integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-a^2*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^2) - 1/3*(b^2*s
in(x)^3 + 3*a*b*sin(x) - 3*b^2*sin(x))/b^3
```

3.29.9 Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx = \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}} - \frac{\sin(x)^3}{3b} - \sin(x)\left(\frac{a+b}{b^2}-\frac{2}{b}\right)$$

input
```
int(cos(x)^5/(a + b*cos(x)^2),x)
```

output
```
(a^2*atanh((b^(1/2)*sin(x))/(a + b)^(1/2)))/(b^(5/2)*(a + b)^(1/2)) - sin(
x)^3/(3*b) - sin(x)*((a + b)/b^2 - 2/b)
```

3.30 $\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx$

3.30.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx=-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sin(x)}{b}$$

output
```
sin(x)/b-a*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx=-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sin(x)}{b}$$

input
```
Integrate[Cos[x]^3/(a + b*Cos[x]^2),x]
```

output
```
-((a*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sin[x
]/b
```

3.30.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3665, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^3}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1-\sin^2(x)}{a-b\sin^2(x)+b}d\sin(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{\sin(x)}{b}-\frac{a\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sin(x)}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

input
```
Int[Cos[x]^3/(a + b*Cos[x]^2),x]
```

output
```
-((a*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sin[x
]/b
```

3.30.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.30.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sin(x)}{b}-\frac{a\,\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}$	33
risch	$-\frac{i\mathrm{e}^{ix}}{2b}+\frac{i\mathrm{e}^{-ix}}{2b}+\frac{a\ln\left(\mathrm{e}^{2ix}-\frac{2i(a+b)\mathrm{e}^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}\,b}-\frac{a\ln\left(\mathrm{e}^{2ix}+\frac{2i(a+b)\mathrm{e}^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}\,b}$	110

input
```
int(cos(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
sin(x)/b-a/b/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.53

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{\sqrt{ab+b^2}a\log\left(-\frac{b\cos(x)^2+2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)+2(ab+b^2)\sin(x)}{2(ab^2+b^3)}, \frac{\sqrt{-ab-b^2}a\arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right)+}{ab^2+b^3}\right.$$

input
```
integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a*b + b^2)*a*log(-(b*cos(x)^2 + 2*sqrt(a*b + b^2)*sin(x) - a -
2*b)/(b*cos(x)^2 + a)) + 2*(a*b + b^2)*sin(x))/(a*b^2 + b^3), (sqrt(-a*b -
 b^2)*a*arctan(sqrt(-a*b - b^2)*sin(x)/(a + b)) + (a*b + b^2)*sin(x))/(a*b
^2 + b^3)]
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx = \frac{a\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}b} + \frac{\sin(x)}{b}$$

input
```
integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*a*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqrt
((a + b)*b)*b) + sin(x)/b
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx = \frac{a\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b} + \frac{\sin(x)}{b}$$

input
```
integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
a*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b) + sin(x)/b
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx = \frac{\sin(x)}{b} - \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

input
```
int(cos(x)^3/(a + b*cos(x)^2),x)
```

output
```
sin(x)/b - (a*atanh((b^(1/2)*sin(x))/(a + b)^(1/2)))/(b^(3/2)*(a + b)^(1/2
))
```

3.31 $\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx$

3.31.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

output
```
arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(1/2)/(a+b)^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

input
```
Integrate[Cos[x]/(a + b*Cos[x]^2),x]
```

output
```
ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

3.31.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3665, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1}{a-b\sin^2(x)+b}d\sin(x)$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

input
```
Int[Cos[x]/(a + b*Cos[x]^2),x]
```

output
```
ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

3.31.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.31.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}}$	21
risch	$\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2i(a+b)\mathrm{e}^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2i(a+b)\mathrm{e}^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}}$	80

input
```
int(cos(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx = \left[\frac{\log\left(-\frac{b\cos(x)^2-2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)}{2\sqrt{ab+b^2}}, -\frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right)}{ab+b^2}\right]$$

input
```
integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(x)^2 - 2*sqrt(a*b + b^2)*sin(x) - a - 2*b)/(b*cos(x)^2 +
a))/sqrt(a*b + b^2), -sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^2)*sin(x)/(a +
 b))/(a*b + b^2)]
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55508 vs. $2(27) = 54$.

Time = 61.71 (sec) , antiderivative size = 55508, normalized size of antiderivative = 1914.07

$$\int \frac{\cos(x)}{a + b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(x)/(a+b*cos(x)**2),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b,
0)), (tan(x/2)/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (sin(x)/a, Eq(b, 0)),
 (-13*a**6*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-sqrt
(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**7*b*sqrt
(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b
) + 2*sqrt(-a*b)/(a + b)) - 130*a**6*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*
sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
24*a**6*b*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*s
qrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 858*a**5*b**3*sqrt(-a
/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) +
 2*sqrt(-a*b)/(a + b)) + 416*a**5*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a +
 b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) - 858*a**4*b**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*
sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 1144*a**4*b**3*sqrt(
-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
+ b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 858*a**3*b**5*sqrt(-a/(a + b) + b/(a
 + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(
a + b)) + 858*a**2*b**6*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b)
)*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 1144*a**2*b**5*sqr
t(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a ...
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\cos(x)}{a + b\cos^2(x)}\,dx = -\frac{\log\left(\frac{b\sin(x) - \sqrt{(a+b)b}}{b\sin(x) + \sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}$$

input
```
integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/sqrt((
a + b)*b)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cos(x)}{a + b\cos^2(x)}\, dx = -\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab - b^2}}$$

input
```
integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-arctan(b*sin(x)/sqrt(-a*b - b^2))/sqrt(-a*b - b^2)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\cos(x)}{a + b\cos^2(x)}\, dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a + b}}$$

input
```
int(cos(x)/(a + b*cos(x)^2),x)
```

output
```
atanh((b^(1/2)*sin(x))/(a + b)^(1/2))/(b^(1/2)*(a + b)^(1/2))
```

3.32 $\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx$

3.32.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx=\frac{\operatorname{arctanh}(\sin(x))}{a}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

output
```
arctanh(sin(x))/a-arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/(a+b)^(1/2
)
```

3.32.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx=\frac{\operatorname{arctanh}(\sin(x))-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a}$$

input
```
Integrate[Sec[x]/(a + b*Cos[x]^2),x]
```

output
```
(ArcTanh[Sin[x]] - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/Sqrt[a
+ b])/a
```

3.32.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3665, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1}{(1-\sin^2(x))\,(a-b\sin^2(x)+b)}d\sin(x)$$

$$\Big\downarrow \text{303}$$

$$\frac{\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a}-\frac{b\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}(\sin(x))}{a}-\frac{b\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}(\sin(x))}{a}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

input
```
Int[Sec[x]/(a + b*Cos[x]^2),x]
```

output
```
ArcTanh[Sin[x]]/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(a*Sqr
t[a + b])
```

3.32.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.32.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{\ln(\sin(x)-1)}{2a}+\frac{\ln(\sin(x)+1)}{2a}-\frac{b\,\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}$	47
risch	$\frac{\ln(\mathrm{e}^{ix}+i)}{a}-\frac{\ln(\mathrm{e}^{ix}-i)}{a}+\frac{\sqrt{(a+b)b}\,\ln\left(\mathrm{e}^{2ix}-\frac{2i\sqrt{(a+b)b}\,\mathrm{e}^{ix}}{b}-1\right)}{2(a+b)a}-\frac{\sqrt{(a+b)b}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{(a+b)b}\,\mathrm{e}^{ix}}{b}-1\right)}{2(a+b)a}$	115

input
```
int(sec(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*ln(sin(x)-1)+1/2/a*ln(sin(x)+1)-b/a/((a+b)*b)^(1/2)*arctanh(b*sin(x
)/((a+b)*b)^(1/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.90

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx = \left[\frac{\sqrt{\frac{b}{a+b}}\log\left(-\frac{b\cos(x)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+\log(\sin(x)+1)-\log(-\sin(x)+1)}{2\,a}, \frac{2\sqrt{-\frac{b}{a+b}}\arctan\left(\right.}{}\right.$$

input
```
integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/(a + b))*log(-(b*cos(x)^2 + 2*(a + b)*sqrt(b/(a + b))*sin(x)
- a - 2*b)/(b*cos(x)^2 + a)) + log(sin(x) + 1) - log(-sin(x) + 1))/a, 1/2*
(2*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x)) + log(sin(x) + 1) - lo
g(-sin(x) + 1))/a]
```

3.32.6 Sympy [F]

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx = \int \frac{\sec(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(sec(x)/(a+b*cos(x)**2),x)
```

output
```
Integral(sec(x)/(a + b*cos(x)**2), x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx = \frac{b\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}a}+\frac{\log(\sin(x)+1)}{2\,a}-\frac{\log(\sin(x)-1)}{2\,a}$$

input
```
integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*b*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqrt
((a + b)*b)*a) + 1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx = \frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a} + \frac{\log\left(\sin\left(x\right)+1\right)}{2\,a} - \frac{\log\left(-\sin\left(x\right)+1\right)}{2\,a}$$

input
```
integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
b*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a) + 1/2*log(sin(x)
+ 1)/a - 1/2*log(-sin(x) + 1)/a
```

3.32.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 414, normalized size of antiderivative = 10.10

$$\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx = \frac{\mathrm{atanh}(\sin\left(x\right))}{a}$$
$$+ \frac{\mathrm{atan}\left(\frac{\frac{\left(2\,b^3\,\sin(x)+\frac{\left(2\,a^2\,b^2-\frac{\sin(x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\sin(x)-\frac{\left(2\,a^2\,b^2+\frac{\sin(x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{b\,(a+}}{4\,\left(a^2+b\,a\right)}\right.}{2\,\left(a^2+b\,a\right)}\right.}{a^2+b\,a}}{\frac{\left(2\,b^3\,\sin(x)+\frac{\left(2\,a^2\,b^2-\frac{\sin(x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\,\sin(x)-\frac{\left(2\,a^2\,b^2+\frac{\sin(x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{b\,(a+}}{4\,\left(a^2+b\,a\right)}\right.}{2\,\left(a^2+b\,a\right)}\right.}{a^2+b\,a}}\right.}{a^2+b\,a}$$

input
```
int(1/(cos(x)*(a + b*cos(x)^2)),x)
```

output
```
atanh(sin(x))/a + (atan((((2*b^3*sin(x) + ((2*a^2*b^2 - (sin(x)*(16*a^2*b^
3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*sin(x) - ((2*a^2*
b^2 + (sin(x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))
*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2))/((
(2*b^3*sin(x) + ((2*a^2*b^2 - (sin(x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))
^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(
1/2))/(a*b + a^2) - ((2*b^3*sin(x) - ((2*a^2*b^2 + (sin(x)*(16*a^2*b^3 + 8
*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b +
 a^2)))*(b*(a + b))^(1/2))/(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2)
```

3.32. $\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx$

3.33 $\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$

3.33.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx = \frac{(a-2b)\text{arctanh}(\sin(x))}{2a^2} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}} + \frac{\sec(x)\tan(x)}{2a}$$

output
```
1/2*(a-2*b)*arctanh(sin(x))/a^2+b^(3/2)*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2)
)/a^2/(a+b)^(1/2)+1/2*sec(x)*tan(x)/a
```

3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 152 vs. 2(59) = 118.

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.58

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$$

$$= \frac{-2(a-2b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(a-2b)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\frac{2b^{3/2}\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)}{\sqrt{a+b}}+\frac{2b^{3/2}\log\left(\right.}{}}{4a^2}$$

input
```
Integrate[Sec[x]^3/(a + b*Cos[x]^2),x]
```

output
```
(-2*(a - 2*b)*Log[Cos[x/2] - Sin[x/2]] + 2*(a - 2*b)*Log[Cos[x/2] + Sin[x/
2]] - (2*b^(3/2)*Log[Sqrt[a + b] - Sqrt[b]*Sin[x]])/Sqrt[a + b] + (2*b^(3/
2)*Log[Sqrt[a + b] + Sqrt[b]*Sin[x]])/Sqrt[a + b] + a/(Cos[x/2] - Sin[x/2]
)^2 - a/(Cos[x/2] + Sin[x/2])^2)/(4*a^2)
```

3.33.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3665, 316, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^3\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(a-b\sin^2(x)+b\right)}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-b\sin^2(x)+a-b}{\left(1-\sin^2(x)\right)\left(-b\sin^2(x)+a+b\right)}d\sin(x)}{2a}+\frac{\sin(x)}{2a\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2b^2\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{a}+\frac{(a-2b)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a}}{2a}+\frac{\sin(x)}{2a\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2b^2\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{a}+\frac{(a-2b)\text{arctanh}(\sin(x))}{a}}{2a}+\frac{\sin(x)}{2a\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-2b)\operatorname{arctanh}(\sin(x))}{a}}{2a}+\frac{\sin(x)}{2a\left(1-\sin^2(x)\right)}$$

input
```
Int[Sec[x]^3/(a + b*Cos[x]^2),x]
```

output
```
(((a - 2*b)*ArcTanh[Sin[x]])/a + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[
a + b]])/(a*Sqrt[a + b]))/(2*a) + Sin[x]/(2*a*(1 - Sin[x]^2))
```

3.33.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.33.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

method	result
default	$-\frac{1}{4a(\sin(x)-1)}+\frac{(-a+2b)\ln(\sin(x)-1)}{4a^2}-\frac{1}{4a(\sin(x)+1)}+\frac{(a-2b)\ln(\sin(x)+1)}{4a^2}+\frac{b^2\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}$
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2a}+\frac{\ln(e^{ix}+i)}{2a}-\frac{\ln(e^{ix}+i)b}{a^2}-\frac{\ln(e^{ix}-i)}{2a}+\frac{\ln(e^{ix}-i)b}{a^2}+\frac{\sqrt{(a+b)b}\,b\ln\left(e^{2ix}+\frac{2i\sqrt{(a+b)b}\,e^{ix}}{b}-1\right)}{2(a+b)a^2}-\frac{\sqrt{(a+b)b}}{}$

input
```
int(sec(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/(sin(x)-1)+1/4/a^2*(-a+2*b)*ln(sin(x)-1)-1/4/a/(sin(x)+1)+1/4*(a-2*
b)/a^2*ln(sin(x)+1)+b^2/a^2/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/
2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.15

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{2\,b\sqrt{\frac{b}{a+b}}\cos(x)^2\log\left(-\frac{b\cos(x)^2-2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+(a-2\,b)\cos(x)^2\log(\sin(x)+1)-(a-2\,b)\,c}{4\,a^2\cos(x)^2}\right.$$
$$-\frac{4\,b\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\sin(x)\right)\cos(x)^2-(a-2\,b)\cos(x)^2\log(\sin(x)+1)+(a-2\,b)\cos(x)^2\log}{4\,a^2\cos(x)^2}$$

input
```
integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(b/(a + b))*cos(x)^2*log(-(b*cos(x)^2 - 2*(a + b)*sqrt(b/(a
+ b))*sin(x) - a - 2*b)/(b*cos(x)^2 + a)) + (a - 2*b)*cos(x)^2*log(sin(x)
+ 1) - (a - 2*b)*cos(x)^2*log(-sin(x) + 1) + 2*a*sin(x))/(a^2*cos(x)^2), -
1/4*(4*b*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x))*cos(x)^2 - (a -
2*b)*cos(x)^2*log(sin(x) + 1) + (a - 2*b)*cos(x)^2*log(-sin(x) + 1) - 2*a*
sin(x))/(a^2*cos(x)^2)]
```

3.33.6 Sympy [F]

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx=\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(sec(x)**3/(a+b*cos(x)**2),x)
```

output
```
Integral(sec(x)**3/(a + b*cos(x)**2), x)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx=-\frac{b^2\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}a^2}+\frac{(a-2\,b)\log\left(\sin(x)+1\right)}{4\,a^2}-\frac{(a-2\,b)\log\left(\sin(x)-1\right)}{4\,a^2}-\frac{\sin(x)}{2\,\left(a\sin(x)^2-a\right)}$$

input
```
integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*b^2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(s
qrt((a + b)*b)*a^2) + 1/4*(a - 2*b)*log(sin(x) + 1)/a^2 - 1/4*(a - 2*b)*lo
g(sin(x) - 1)/a^2 - 1/2*sin(x)/(a*sin(x)^2 - a)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx = -\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2} + \frac{(a-2b)\log(\sin(x)+1)}{4a^2} - \frac{(a-2b)\log(-\sin(x)+1)}{4a^2} - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)a}$$

input
```
integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-b^2*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2) + 1/4*(a - 2
*b)*log(sin(x) + 1)/a^2 - 1/4*(a - 2*b)*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/
((sin(x)^2 - 1)*a)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 483, normalized size of antiderivative = 8.19

$$\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx =$$

$$-\frac{a^2\sin(x)+a^2\operatorname{atanh}(\sin(x))-2b^2\operatorname{atanh}(\sin(x))+ab\sin(x)-ab\operatorname{atanh}(\sin(x))-a^2\operatorname{atanh}(\sin(x)}{}$$

input
```
int(1/(cos(x)^3*(a + b*cos(x)^2)),x)
```

output
```
-(a^2*sin(x) + a^2*atanh(sin(x)) - 2*b^2*atanh(sin(x)) + atan((b^5*sin(x)*
(a*b^3 + b^4)^(1/2)*8i - a*sin(x)*(a*b^3 + b^4)^(3/2)*4i - b*sin(x)*(a*b^3
 + b^4)^(3/2)*8i + a*b^4*sin(x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b*sin(x)*(a*
b^3 + b^4)^(1/2)*1i + a^2*b^3*sin(x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*sin(
x)*(a*b^3 + b^4)^(1/2)*2i)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^5*b^2))*(a
*b^3 + b^4)^(1/2)*2i + a*b*sin(x) - a*b*atanh(sin(x)) - a^2*atanh(sin(x))*
sin(x)^2 + 2*b^2*atanh(sin(x))*sin(x)^2 - atan((b^5*sin(x)*(a*b^3 + b^4)^(
1/2)*8i - a*sin(x)*(a*b^3 + b^4)^(3/2)*4i - b*sin(x)*(a*b^3 + b^4)^(3/2)*8
i + a*b^4*sin(x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b*sin(x)*(a*b^3 + b^4)^(1/2
)*1i + a^2*b^3*sin(x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*sin(x)*(a*b^3 + b^4
)^(1/2)*2i)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^5*b^2))*sin(x)^2*(a*b^3 +
 b^4)^(1/2)*2i + a*b*atanh(sin(x))*sin(x)^2)/(2*a^3*sin(x)^2 - 2*a^2*b - 2
*a^3 + 2*a^2*b*sin(x)^2)
```

3.34 $\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx$

3.34.1 Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx=\frac{(3a^2-4ab+8b^2)\,\text{arctanh}(\sin(x))}{8a^3}-\frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a}$$

output
```
1/8*(3*a^2-4*a*b+8*b^2)*arctanh(sin(x))/a^3-b^(5/2)*arctanh(sin(x)*b^(1/2)
/(a+b)^(1/2))/a^3/(a+b)^(1/2)+1/8*(3*a-4*b)*sec(x)*tan(x)/a^2+1/4*sec(x)^3
*tan(x)/a
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(90) = 180.

Time = 0.85 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.39

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx$$
$$=\frac{-2(3a^2-4ab+8b^2)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(3a^2-4ab+8b^2)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{8b^{5/2}\log\left(\sqrt{a+b}-\right.}{\sqrt{a+b}}}{16a^3}$$

input
```
Integrate[Sec[x]^5/(a + b*Cos[x]^2),x]
```

output
```
(-2*(3*a^2 - 4*a*b + 8*b^2)*Log[Cos[x/2] - Sin[x/2]] + 2*(3*a^2 - 4*a*b +
8*b^2)*Log[Cos[x/2] + Sin[x/2]] + (8*b^(5/2)*Log[Sqrt[a + b] - Sqrt[b]*Sin
[x]])/Sqrt[a + b] - (8*b^(5/2)*Log[Sqrt[a + b] + Sqrt[b]*Sin[x]])/Sqrt[a +
 b] + a^2/(Cos[x/2] - Sin[x/2])^4 - a^2/(Cos[x/2] + Sin[x/2])^4 + (a*(-3*a
 + 4*b))/(Cos[x/2] + Sin[x/2])^2 + (a*(-3*a + 4*b))/(-1 + Sin[x]))/(16*a^3
)
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3665, 316, 402, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^5\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(a-b\sin^2(x)+b\right)}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-3b\sin^2(x)+3a-b}{\left(1-\sin^2(x)\right)^2\left(-b\sin^2(x)+a+b\right)}d\sin(x)}{4a}+\frac{\sin(x)}{4a\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{3a^2-ba+4b^2-(3a-4b)b\sin^2(x)}{\left(1-\sin^2(x)\right)\left(-b\sin^2(x)+a+b\right)}d\sin(x)}{2a}+\frac{(3a-4b)\sin(x)}{2a\left(1-\sin^2(x)\right)}}{4a}+\frac{\sin(x)}{4a\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a}-\frac{8b^3\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{a}}{2a}+\frac{(3a-4b)\sin(x)}{2a(1-\sin^2(x))}}{4a}+\frac{\sin(x)}{4a\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\text{arctanh}(\sin(x))}{a}-\frac{8b^3\int \frac{1}{-b\sin^2(x)+a+b}d\sin(x)}{a}}{2a}+\frac{(3a-4b)\sin(x)}{2a(1-\sin^2(x))}}{4a}+\frac{\sin(x)}{4a\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\text{arctanh}(\sin(x))}{a}-\frac{8b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}+\frac{(3a-4b)\sin(x)}{2a(1-\sin^2(x))}}{4a}+\frac{\sin(x)}{4a\left(1-\sin^2(x)\right)^2}$$

input
```
Int[Sec[x]^5/(a + b*Cos[x]^2),x]
```

output
```
Sin[x]/(4*a*(1 - Sin[x]^2)^2) + ((((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Sin[x]]
)/a - (8*b^(5/2)*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(
2*a) + ((3*a - 4*b)*Sin[x])/(2*a*(1 - Sin[x]^2)))/(4*a)
```

3.34.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.34.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.52

method	result
default	$\frac{1}{16a(\sin(x)-1)^2} - \frac{3a-4b}{16a^2(\sin(x)-1)} + \frac{(-3a^2+4ab-8b^2)\ln(\sin(x)-1)}{16a^3} - \frac{1}{16a(\sin(x)+1)^2} - \frac{3a-4b}{16a^2(\sin(x)+1)} + \frac{(3a^2-4ab+8b^2}{16}$
risch	$-\frac{i(3a\,e^{7ix}-4b\,e^{7ix}+11a\,e^{5ix}-4b\,e^{5ix}-11a\,e^{3ix}+4b\,e^{3ix}-3\,e^{ix}a+4\,e^{ix}b)}{4(e^{2ix}+1)^4a^2} + \frac{3\ln(e^{ix}+i)}{8a} - \frac{\ln(e^{ix}+i)b}{2a^2} + \frac{\ln(e^{ix}+i)b^2}{a^3} - \frac{3\ln(e}{8}$

input
```
int(sec(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a/(sin(x)-1)^2-1/16*(3*a-4*b)/a^2/(sin(x)-1)+1/16/a^3*(-3*a^2+4*a*b-8
*b^2)*ln(sin(x)-1)-1/16/a/(sin(x)+1)^2-1/16*(3*a-4*b)/a^2/(sin(x)+1)+1/16*
(3*a^2-4*a*b+8*b^2)/a^3*ln(sin(x)+1)-b^3/a^3/((a+b)*b)^(1/2)*arctanh(b*sin
(x)/((a+b)*b)^(1/2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.00

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{8\,b^2\sqrt{\frac{b}{a+b}}\cos(x)^4\log\left(-\frac{b\cos(x)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+(3\,a^2-4\,ab+8\,b^2)\cos(x)^4\log(\sin(x)+1)-}{16\,a^3\cos(x)^4} \right.$$

input
```
integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/16*(8*b^2*sqrt(b/(a + b))*cos(x)^4*log(-(b*cos(x)^2 + 2*(a + b)*sqrt(b/
(a + b))*sin(x) - a - 2*b)/(b*cos(x)^2 + a)) + (3*a^2 - 4*a*b + 8*b^2)*cos
(x)^4*log(sin(x) + 1) - (3*a^2 - 4*a*b + 8*b^2)*cos(x)^4*log(-sin(x) + 1)
+ 2*((3*a^2 - 4*a*b)*cos(x)^2 + 2*a^2)*sin(x))/(a^3*cos(x)^4), 1/16*(16*b^
2*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x))*cos(x)^4 + (3*a^2 - 4*a
*b + 8*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a^2 - 4*a*b + 8*b^2)*cos(x)^4*lo
g(-sin(x) + 1) + 2*((3*a^2 - 4*a*b)*cos(x)^2 + 2*a^2)*sin(x))/(a^3*cos(x)^
4)]
```

3.34.6 Sympy [F]

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx = \int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(sec(x)**5/(a+b*cos(x)**2),x)
```

output
```
Integral(sec(x)**5/(a + b*cos(x)**2), x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.61

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx = \frac{b^3\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}a^3} - \frac{(3\,a-4\,b)\sin(x)^3-(5\,a-4\,b)\sin(x)}{8\left(a^2\sin(x)^4-2\,a^2\sin(x)^2+a^2\right)} + \frac{(3\,a^2-4\,ab+8\,b^2)\log(\sin(x)+1)}{16\,a^3} - \frac{(3\,a^2-4\,ab+8\,b^2)\log(\sin(x)-1)}{16\,a^3}$$

input
```
integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*b^3*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sq
rt((a + b)*b)*a^3) - 1/8*((3*a - 4*b)*sin(x)^3 - (5*a - 4*b)*sin(x))/(a^2*
sin(x)^4 - 2*a^2*sin(x)^2 + a^2) + 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(sin(x)
 + 1)/a^3 - 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(sin(x) - 1)/a^3
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.41

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx = \frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^3} + \frac{(3\,a^2-4\,ab+8\,b^2)\log(\sin(x)+1)}{16\,a^3} - \frac{(3\,a^2-4\,ab+8\,b^2)\log(-\sin(x)+1)}{16\,a^3} - \frac{3\,a\sin(x)^3-4\,b\sin(x)^3-5\,a\sin(x)+4\,b\sin(x)}{8\left(\sin(x)^2-1\right)^2a^2}$$

input
```
integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
b^3*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^3) + 1/16*(3*a^2
 - 4*a*b + 8*b^2)*log(sin(x) + 1)/a^3 - 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(-
sin(x) + 1)/a^3 - 1/8*(3*a*sin(x)^3 - 4*b*sin(x)^3 - 5*a*sin(x) + 4*b*sin(
x))/((sin(x)^2 - 1)^2*a^2)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 969, normalized size of antiderivative = 10.77

$$\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(x)^5*(a + b*cos(x)^2)),x)
```

output
```
(5*a^3*sin(x) + atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i - a*sin(x)*(a*b^
5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b^6*sin(x)*(a*b
^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i + a^2*b^5*sin(x
)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)*40i + a^4*b
^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6)^(1/2)*6i)
/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))*(a*b^5 +
b^6)^(1/2)*8i - 3*a^3*sin(x)^3 + 3*a^3*atanh(sin(x)) + 8*b^3*atanh(sin(x))
 - 4*a*b^2*sin(x) + a^2*b*sin(x) - atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*12
8i - a*sin(x)*(a*b^5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i
+ a*b^6*sin(x)*(a*b^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)
*9i + a^2*b^5*sin(x)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6
)^(1/2)*40i + a^4*b^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b
^5 + b^6)^(1/2)*6i)/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*
a^7*b^3))*sin(x)^2*(a*b^5 + b^6)^(1/2)*16i + atan((b^7*sin(x)*(a*b^5 + b^6
)^(1/2)*128i - a*sin(x)*(a*b^5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(
3/2)*128i + a*b^6*sin(x)*(a*b^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 +
b^6)^(1/2)*9i + a^2*b^5*sin(x)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a
*b^5 + b^6)^(1/2)*40i + a^4*b^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*s
in(x)*(a*b^5 + b^6)^(1/2)*6i)/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^
6*b^4 + 9*a^7*b^3))*sin(x)^4*(a*b^5 + b^6)^(1/2)*8i - 6*a^3*atanh(sin(x...
```

3.35 $\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx$

3.35.1 Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx = \frac{(8a^2-4ab+3b^2)\,x}{8b^3} + \frac{a^{5/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}} - \frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(x)\sin(x)}{4b}$$

output
```
1/8*(8*a^2-4*a*b+3*b^2)*x/b^3-1/8*(4*a-3*b)*cos(x)*sin(x)/b^2+1/4*cos(x)^3
*sin(x)/b+a^(5/2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/b^3/(a+b)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx = \frac{4(8a^2-4ab+3b^2)\,x - \frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - 8(a-b)b\sin(2x) + b^2\sin(4x)}{32b^3}$$

input
```
Integrate[Cos[x]^6/(a + b*Cos[x]^2),x]
```

output
```
(4*(8*a^2 - 4*a*b + 3*b^2)*x - (32*a^(5/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a
+ b]])/Sqrt[a + b] - 8*(a - b)*b*Sin[2*x] + b^2*Sin[4*x])/(32*b^3)
```

3.35.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3666, 372, 440, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^6}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 3666$$

$$-\int \frac{\cot^6(x)}{\left(\cot^2(x)+1\right)^3\left((a+b)\cot^2(x)+a\right)}d\cot(x)$$

$$\Big\downarrow \ 372$$

$$\frac{\cot^3(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\int \frac{\cot^2(x)\left(3a-(a-3b)\cot^2(x)\right)}{\left(\cot^2(x)+1\right)^2\left((a+b)\cot^2(x)+a\right)}d\cot(x)}{4b}$$

$$\Big\downarrow \ 440$$

$$\frac{\cot^3(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{(4a-3b)\cot(x)}{2b(\cot^2(x)+1)}-\frac{\int \frac{a(4a-3b)-\left(4a^2-ba+3b^2\right)\cot^2(x)}{\left(\cot^2(x)+1\right)\left((a+b)\cot^2(x)+a\right)}d\cot(x)}{2b}}{4b}$$

$$\Big\downarrow \ 397$$

$$\frac{\cot^3(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{(4a-3b)\cot(x)}{2b(\cot^2(x)+1)}-\frac{\frac{8a^3\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{\left(8a^2-4ab+3b^2\right)\int \frac{1}{\cot^2(x)+1}d\cot(x)}{b}}{2b}}{4b}$$

$$\Big\downarrow \ 216$$

$$\frac{\cot^3(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{(4a-3b)\cot(x)}{2b(\cot^2(x)+1)}-\frac{\frac{8a^3\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{\left(8a^2-4ab+3b^2\right)\arctan(\cot(x))}{b}}{2b}}{4b}$$

$$\Big\downarrow \ 218$$

$$\frac{\cot^3(x)}{4b\left(\cot^2(x)+1\right)^2}-\frac{\frac{(4a-3b)\cot(x)}{2b(\cot^2(x)+1)}-\frac{\frac{8a^{5/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{\left(8a^2-4ab+3b^2\right)\arctan(\cot(x))}{b}}{2b}}{4b}$$

input
```
Int[Cos[x]^6/(a + b*Cos[x]^2),x]
```

output
```
Cot[x]^3/(4*b*(1 + Cot[x]^2)^2) - (-1/2*(-(((8*a^2 - 4*a*b + 3*b^2)*ArcTan
[Cot[x]])/b) + (8*a^(5/2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(b*Sqrt[a
+ b]))/b + ((4*a - 3*b)*Cot[x])/(2*b*(1 + Cot[x]^2)))/(4*b)
```

3.35.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.35.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

method	result
default	$-\frac{a^{3} \arctan\left(\frac{a \tan(x)}{\sqrt{(a+b)a}}\right)}{b^{3} \sqrt{(a+b)a}}+\frac{\frac{\left(-\frac{1}{2} a b+\frac{3}{8} b^{2}\right)\left(\tan^{3}(x)\right)+\left(-\frac{1}{2} a b+\frac{5}{8} b^{2}\right) \tan(x)}{\left(\tan^{2}(x)+1\right)^{2}}+\frac{\left(8 a^{2}-4 a b+3 b^{2}\right) \arctan(\tan(x))}{8}}{b^{3}}$
risch	$\frac{x a^{2}}{b^{3}}-\frac{x a}{2 b^{2}}+\frac{3 x}{8 b}+\frac{i e^{2 i x} a}{8 b^{2}}-\frac{i e^{2 i x}}{8 b}-\frac{i e^{-2 i x} a}{8 b^{2}}+\frac{i e^{-2 i x}}{8 b}+\frac{\sqrt{-(a+b) a}\, a^{2} \ln\left(e^{2 i x}+\frac{2 i \sqrt{-(a+b) a}+2 a+b}{b}\right)}{2(a+b) b^{3}}-\frac{\sqrt{-(a+b) a}\, a^{2}}{}$

input
```
int(cos(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*a^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))+1/b^3*(((-1/2*
a*b+3/8*b^2)*tan(x)^3+(-1/2*a*b+5/8*b^2)*tan(x))/(tan(x)^2+1)^2+1/8*(8*a^2
-4*a*b+3*b^2)*arctan(tan(x)))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.14

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{2a^2\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a^2+3ab+b^2)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)+}{8b^3} \right.$$

input
```
integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^
2 + 3*a*b)*cos(x)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a^2 + a*b)*cos(
x))*sqrt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))
+ (8*a^2 - 4*a*b + 3*b^2)*x + (2*b^2*cos(x)^3 - (4*a*b - 3*b^2)*cos(x))*si
n(x))/b^3, 1/8*(4*a^2*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*
sqrt(a/(a + b))/(a*cos(x)*sin(x))) + (8*a^2 - 4*a*b + 3*b^2)*x + (2*b^2*co
s(x)^3 - (4*a*b - 3*b^2)*cos(x))*sin(x))/b^3]
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**6/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx = -\frac{a^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^3} - \frac{(4a-3b)\tan(x)^3+(4a-5b)\tan(x)}{8\left(b^2\tan(x)^4+2b^2\tan(x)^2+b^2\right)} + \frac{(8a^2-4ab+3b^2)x}{8b^3}$$

input
```
integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-a^3*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3) - 1/8*((4*a -
3*b)*tan(x)^3 + (4*a - 5*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(x)^2 + b^2)
+ 1/8*(8*a^2 - 4*a*b + 3*b^2)*x/b^3
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx = -\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a^3}{\sqrt{a^2+ab}b^3} + \frac{(8a^2-4ab+3b^2)x}{8b^3} - \frac{4a\tan(x)^3-3b\tan(x)^3+4a\tan(x)-5b\tan(x)}{8\left(\tan(x)^2+1\right)^2b^2}$$

input
```
integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a^3/(sqr
t(a^2 + a*b)*b^3) + 1/8*(8*a^2 - 4*a*b + 3*b^2)*x/b^3 - 1/8*(4*a*tan(x)^3
- 3*b*tan(x)^3 + 4*a*tan(x) - 5*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 1036, normalized size of antiderivative = 11.91

$$\int \frac{\cos^6(x)}{a + b\cos^2(x)}\, dx = \text{Too large to display}$$

input
```
int(cos(x)^6/(a + b*cos(x)^2),x)
```

output
```
- ((tan(x)^3*(4*a - 3*b))/(8*b^2) + (tan(x)*(4*a - 5*b))/(8*b^2))/(2*tan(x
)^2 + tan(x)^4 + 1) - (atan((63*a^4*tan(x))/(64*((63*a^4)/64 - (81*a^3*b)/
256 + (27*a^2*b^2)/256 - (35*a^5)/(32*b) + (5*a^6)/(4*b^2))) - (81*a^3*tan
(x))/(256*((27*a^2*b)/256 - (81*a^3)/256 + (63*a^4)/(64*b) - (35*a^5)/(32*
b^2) + (5*a^6)/(4*b^3))) - (35*a^5*tan(x))/(32*((63*a^4*b)/64 - (35*a^5)/3
2 + (27*a^2*b^3)/256 - (81*a^3*b^2)/256 + (5*a^6)/(4*b))) + (5*a^6*tan(x))
/(4*((5*a^6)/4 - (35*a^5*b)/32 + (27*a^2*b^4)/256 - (81*a^3*b^3)/256 + (63
*a^4*b^2)/64)) + (27*a^2*tan(x))/(256*((27*a^2)/256 - (81*a^3)/(256*b) + (
63*a^4)/(64*b^2) - (35*a^5)/(32*b^3) + (5*a^6)/(4*b^4))))*(a^2*8i - a*b*4i
 + b^2*3i)*1i)/(8*b^3) - (atan((((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/
2)*(((3*a^2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(256*a^2*b
^7 + 512*a^3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^
3 + b^4)) - (tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*
b^2))/(64*b^4))*1i)/(a*b^3 + b^4) - ((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))
^(1/2)*(((3*a^2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(256*a
^2*b^7 + 512*a^3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(
a*b^3 + b^4)) + (tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*
a^5*b^2))/(64*b^4))*1i)/(a*b^3 + b^4))/(((-a^5*(a + b))^(1/2)*(((-a^5*(a +
 b))^(1/2)*(((3*a^2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(2
56*a^2*b^7 + 512*a^3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))...
```

3.36 $\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx$

3.36.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx = -\frac{(2a-b)x}{2b^2} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}} + \frac{\cos(x)\sin(x)}{2b}$$

output
```
-1/2*(2*a-b)*x/b^2+1/2*cos(x)*sin(x)/b-a^(3/2)*arctan(cot(x)*(a+b)^(1/2)/a
^(1/2))/b^2/(a+b)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx = \frac{2(-2a+b)x + \frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + b\sin(2x)}{4b^2}$$

input
```
Integrate[Cos[x]^4/(a + b*Cos[x]^2),x]
```

output
```
(2*(-2*a + b)*x + (4*a^(3/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a
+ b] + b*Sin[2*x])/(4*b^2)
```

3.36.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3666, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^4}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$-\int \frac{\cot^4(x)}{\left(\cot^2(x)+1\right)^2\left((a+b)\cot^2(x)+a\right)}d\cot(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\int \frac{a-(a-b)\cot^2(x)}{\left(\cot^2(x)+1\right)\left((a+b)\cot^2(x)+a\right)}d\cot(x)}{2b}$$

$$\Big\downarrow \text{397}$$

$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\frac{2a^2\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{(2a-b)\int \frac{1}{\cot^2(x)+1}d\cot(x)}{b}}{2b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\frac{2a^2\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}-\frac{(2a-b)\arctan(\cot(x))}{b}}{2b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\cot(x)}{2b\left(\cot^2(x)+1\right)}-\frac{\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{(2a-b)\arctan(\cot(x))}{b}}{2b}$$

input
```
Int[Cos[x]^4/(a + b*Cos[x]^2),x]
```

output
```
-1/2*(-(((2*a - b)*ArcTan[Cot[x]])/b) + (2*a^(3/2)*ArcTan[(Sqrt[a + b]*Cot
[x])/Sqrt[a]])/(b*Sqrt[a + b]))/b + Cot[x]/(2*b*(1 + Cot[x]^2))
```

3.36.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.36. $\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx$

3.36.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result
default	$\frac{a^2 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{b^2\sqrt{(a+b)a}}-\frac{-\frac{b\tan(x)}{2\left(\tan^2(x)+1\right)}+\frac{(2a-b)\arctan(\tan(x))}{2}}{b^2}$
risch	$-\frac{xa}{b^2}+\frac{x}{2b}-\frac{i e^{2ix}}{8b}+\frac{i e^{-2ix}}{8b}+\frac{\sqrt{-(a+b)a}\, a\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)a}-2a-b}{b}\right)}{2(a+b)b^2}-\frac{\sqrt{-(a+b)a}\, a\ln\left(e^{2ix}+\frac{2i\sqrt{-(a+b)a}+2a+b}{b}\right)}{2(a+b)b^2}$

input
```
int(cos(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
a^2/b^2/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))-1/b^2*(-1/2*b*tan
(x)/(tan(x)^2+1)+1/2*(2*a-b)*arctan(tan(x)))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.55

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx$$
$$=\left[\frac{2\,b\cos(x)\sin(x)+a\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2-4\,\left((2\,a^2+3\,ab+b^2)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{}}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right.}{4\,b^2}\right.$$

input
```
integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*cos(x)*sin(x) + a*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*co
s(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 -
(a^2 + a*b)*cos(x))*sqrt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*c
os(x)^2 + a^2)) - 2*(2*a - b)*x)/b^2, 1/2*(b*cos(x)*sin(x) - a*sqrt(a/(a +
 b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(a/(a + b))/(a*cos(x)*sin(x))
) - (2*a - b)*x)/b^2]
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**4/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx = \frac{a^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^2} - \frac{(2a-b)x}{2b^2} + \frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}$$

input
```
integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
a^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2) - 1/2*(2*a - b)
*x/b^2 + 1/2*tan(x)/(b*tan(x)^2 + b)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}b^2} - \frac{(2a-b)x}{2b^2} + \frac{\tan(x)}{2\left(\tan(x)^2+1\right)b}$$

input
```
integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a^2/(sqrt
(a^2 + a*b)*b^2) - 1/2*(2*a - b)*x/b^2 + 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 291, normalized size of antiderivative = 4.85

$$\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx =$$

$$-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-b^2\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{b^2\,\sin(2\,x)}{2}+a\,b\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{a\,b\,\sin(2\,x)}{2}+\mathrm{atan}\left(\frac{a\,\sin(x)\,\left(-a^4-b\,a^3\right)^{3/2}}{\right.}}{}$$

input
```
int(cos(x)^4/(a + b*cos(x)^2),x)
```

output
```
-(2*a^2*atan(sin(x)/cos(x)) - b^2*atan(sin(x)/cos(x)) + atan((a*sin(x)*(-
a^3*b - a^4)^(3/2)*8i + b*sin(x)*(- a^3*b - a^4)^(3/2)*4i + a^5*sin(x)*(-
a^3*b - a^4)^(1/2)*8i - a^2*b^3*sin(x)*(- a^3*b - a^4)^(1/2)*2i + a^3*b^2*
sin(x)*(- a^3*b - a^4)^(1/2)*1i + a*b^4*sin(x)*(- a^3*b - a^4)^(1/2)*1i +
a^4*b*sin(x)*(- a^3*b - a^4)^(1/2)*12i)/(a^3*b^4*cos(x) - a^2*b^5*cos(x) +
 5*a^4*b^3*cos(x) + 3*a^5*b^2*cos(x)))*(- a^3*b - a^4)^(1/2)*2i - (b^2*sin
(2*x))/2 + a*b*atan(sin(x)/cos(x)) - (a*b*sin(2*x))/2)/(2*a*b^2 + 2*b^3)
```

3.37 $\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx$

3.37.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = \frac{x}{b}+\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}$$

output
```
x/b+arctan(cot(x)*(a+b)^(1/2)/a^(1/2))*a^(1/2)/b/(a+b)^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = \frac{x-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{b}$$

input
```
Integrate[Cos[x]^2/(a + b*Cos[x]^2),x]
```

output
```
(x - (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a + b])/b
```

3.37.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3650, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^2}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3650}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\cos^2(x)+a}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{b}$$

$$\Big\downarrow \text{3660}$$

$$\frac{a\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{b}+\frac{x}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}+\frac{x}{b}$$

input
```
Int[Cos[x]^2/(a + b*Cos[x]^2),x]
```

output
```
x/b + (Sqrt[a]*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(b*Sqrt[a + b])
```

3.37.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.37.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a \arctan\left(\frac{a \tan(x)}{\sqrt{(a+b)a}}\right)}{b\sqrt{(a+b)a}}+\frac{\arctan(\tan(x))}{b}$	34
risch	$\frac{x}{b}+\frac{\sqrt{-(a+b)a}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-(a+b)a}+2a+b}{b}\right)}{2(a+b)b}-\frac{\sqrt{-(a+b)a}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)a}-2a-b}{b}\right)}{2(a+b)b}$	100

input
```
int(cos(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-a/b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))+1/b*arctan(tan(x))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 4.82

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a^2+3ab+b^2)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)+4x}{4b}\right.$$

input
```
integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a^2 + a*b)*cos(x))*sq
rt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) + 4*x)
/b, 1/2*(sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(a/(a + b
))/(a*cos(x)*sin(x))) + 2*x)/b]
```

3.37.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2/(a+b*cos(x)**2),x)
```

output
```
Timed out
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = -\frac{a\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b}+\frac{x}{b}$$

input
```
integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-a*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) + x/b
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = -\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}b}+\frac{x}{b}$$

input
```
integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a/(sqrt(
a^2 + a*b)*b) + x/b
```

3.37.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 425, normalized size of antiderivative = 11.18

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = \frac{x}{b}$$
$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,a^3\,\tan(x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}\,1i}{b^2+a\,b}+\frac{\left(2\,a^3\,\tan(x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)}{4\,\left(b^2+a\,b\right)}\cdots}{2\,\left(b^2+a\,b\right)}\cdots}{b^2+a\,b}}{\frac{\left(2\,a^3\,\tan(x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,(a+b)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\tan(x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)}{4\,\left(b^2+a\,b\right)}\cdots}{2\,\left(b^2+a\,b\right)}\cdots}{b^2+a\,b}}\cdots\right.}{b^2+a\,b\cdots}$$

input
```
int(cos(x)^2/(a + b*cos(x)^2),x)
```

output
```
x/b - (atan((((2*a^3*tan(x) - ((2*a^2*b^2 - (tan(x)*(8*a^2*b^3 + 16*a^3*b^
2)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)
))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2*a^3*tan(x) + ((2*a^2*b^2 + (ta
n(x)*(8*a^2*b^3 + 16*a^3*b^2)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a
+ b))^(1/2))/(2*(a*b + b^2)))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3
*tan(x) - ((2*a^2*b^2 - (tan(x)*(8*a^2*b^3 + 16*a^3*b^2)*(-a*(a + b))^(1/2
))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))*(-a*(a + b))^(1/2
))/(a*b + b^2) - ((2*a^3*tan(x) + ((2*a^2*b^2 + (tan(x)*(8*a^2*b^3 + 16*a^
3*b^2)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b +
b^2)))*(-a*(a + b))^(1/2))/(a*b + b^2)))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2
)
```

3.37. $\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx$

3.38 $\int \frac{1}{a+b\cos^2(x)}\,dx$

3.38.1 Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{a+b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
-arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

3.38.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{3660}$$
$$-\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)$$
$$\Big\downarrow \text{218}$$
$$-\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

3.38.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.38.4 Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$	21
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}+\frac{\ln\left(e^{2ix}+\frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}$	158

input
```
int(1/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.43

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{4(a^2+ab)}, -\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2
*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)
]
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(29) = 58$.

Time = 20.82 (sec) , antiderivative size = 10924, normalized size of antiderivative = 364.13

$$\int \frac{1}{a + b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-tan(x/2
)/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (-2*tan(x/2)/(b*(tan(x/2)**2 - 1))
, Eq(a, 0)), (a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log
(-sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*
sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a
 + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*
sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt
(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a
+ b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*s
qrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(
a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqr
t(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b
) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-
a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) -
2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b...
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{a + b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{1}{a + b\cos^2(x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2
+ a*b)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{a + b\cos^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*cos(x)^2),x)
```

output
```
atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

3.39 $\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx$

3.39.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{\tan(x)}{a}$$

output
```
b*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(1/2)+tan(x)/a
```

3.39.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx = -\frac{b\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{\tan(x)}{a}$$

input
```
Integrate[Sec[x]^2/(a + b*Cos[x]^2),x]
```

output
```
-((b*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b])) + Tan[x]
/a
```

3.39.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3666, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$-\int \frac{\left(\cot^2(x)+1\right)\tan^2(x)}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{359}$$

$$\frac{b\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{a}+\frac{\tan(x)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{a}$$

input
```
Int[Sec[x]^2/(a + b*Cos[x]^2),x]
```

output
```
(b*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]) + Tan[x]/a
```

3.39.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.39.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\tan(x)}{a}-\frac{b \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{a\sqrt{(a+b)a}}$	33
risch	$\frac{2i}{(e^{2ix}+1)a}-\frac{b\ln\left(e^{2ix}+\frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,a}+\frac{b\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,a}$	181

input
```
int(sec(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/a-b/a/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 5.84

$$\int \frac{\sec^2(x)}{a + b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2-ab}b\cos(x)\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2-4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right) -}{4(a^3+a^2b)\cos(x)}\right.$$

input
```
integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(
4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*
b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*(a^2 + a*b)*si
n(x))/((a^3 + a^2*b)*cos(x)), 1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)
*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) + 2*(a^2 + a*b)*sin
(x))/((a^3 + a^2*b)*cos(x))]
```

3.39.6 Sympy [F]

$$\int \frac{\sec^2(x)}{a + b\cos^2(x)}\,dx = \int \frac{\sec^2(x)}{a + b\cos^2(x)}\,dx$$

input
```
integrate(sec(x)**2/(a+b*cos(x)**2),x)
```

output
```
Integral(sec(x)**2/(a + b*cos(x)**2), x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx = -\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a} + \frac{\tan(x)}{a}$$

input
```
integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-b*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a) + tan(x)/a
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx = -\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}a} + \frac{\tan(x)}{a}$$

input
```
integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-b*arctan(a*tan(x)/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*a) + tan(x)/a
```

3.39.9 Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx = \frac{\tan(x)}{a} - \frac{b\,\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}$$

input
```
int(1/(cos(x)^2*(a + b*cos(x)^2)),x)
```

output
```
tan(x)/a - (b*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(3/2)*(a + b)^(1/2)
)
```

3.40 $\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx$

3.40.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx=-\frac{b^2\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}$$

output
```
-b^2*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(1/2)+(a-b)*tan(x)/a
^2+1/3*tan(x)^3/a
```

3.40.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(2a-3b+a\sec^2(x))\tan(x)}{3a^2}$$

input
```
Integrate[Sec[x]^4/(a + b*Cos[x]^2),x]
```

output
```
(b^2*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + ((2*a -
 3*b + a*Sec[x]^2)*Tan[x])/(3*a^2)
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^4\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \ 3666$$

$$-\int \frac{\left(\cot^2(x)+1\right)^2\tan^4(x)}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \ 364$$

$$-\int \left(\frac{\tan^4(x)}{a}+\frac{(a-b)\tan^2(x)}{a^2}+\frac{b^2}{a^2\left((a+b)\cot^2(x)+a\right)}\right)d\cot(x)$$

$$\Big\downarrow \ 2009$$

$$-\frac{b^2\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}$$

input
```
Int[Sec[x]^4/(a + b*Cos[x]^2),x]
```

output
```
-((b^2*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b])) + ((a
- b)*Tan[x])/a^2 + Tan[x]^3/(3*a)
```

3.40.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.40.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{a\left(\tan^{3}(x)\right)}{3}+\tan(x)a-\tan(x)b}{a^{2}}+\frac{b^{2}\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{a^{2}\sqrt{(a+b)a}}$
risch	$-\frac{2i\left(3b\,e^{4ix}-6a\,e^{2ix}+6b\,e^{2ix}-2a+3b\right)}{3\left(e^{2ix}+1\right)^{3}a^{2}}-\frac{b^{2}\ln\left(e^{2ix}+\frac{2ia^{2}+2iab+2a\sqrt{-a^{2}-ab}+b\sqrt{-a^{2}-ab}}{b\sqrt{-a^{2}-ab}}\right)}{2\sqrt{-a^{2}-ab}\,a^{2}}+\frac{b^{2}\ln\left(e^{2ix}+\frac{-2ia^{2}-2iab+2a\sqrt{-a^{2}-}}{b\sqrt{-a^{2}-a}}\right.}{2\sqrt{-a^{2}-ab}\,a^{2}}$

input
```
int(sec(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/3*a*tan(x)^3+tan(x)*a-tan(x)*b)+b^2/a^2/((a+b)*a)^(1/2)*arctan(a*
tan(x)/((a+b)*a)^(1/2))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 4.93

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx$$
$$= \left[-\frac{3\sqrt{-a^2-ab}b^2\cos(x)^3\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{12(a^4+a^3b)\cos(x)^3}\right.$$
$$\left.-\frac{3\sqrt{a^2+ab}b^2\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\cos(x)^3-2\left(a^3+a^2b+(2a^3-a^2b-3ab^2)\cos(x)^2\right)\sin(x)}{6(a^4+a^3b)\cos(x)^3}\right]$$

input
```
integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a
^2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*(a^3 +
a^2*b + (2*a^3 - a^2*b - 3*a*b^2)*cos(x)^2)*sin(x))/((a^4 + a^3*b)*cos(x)^
3), -1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(
a^2 + a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + a^2*b + (2*a^3 - a^2*b - 3*
a*b^2)*cos(x)^2)*sin(x))/((a^4 + a^3*b)*cos(x)^3)]
```

3.40.6 Sympy [F]

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx = \int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx$$

input
```
integrate(sec(x)**4/(a+b*cos(x)**2),x)
```

output
```
Integral(sec(x)**4/(a + b*cos(x)**2), x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx = \frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^2} + \frac{a\tan(x)^3+3(a-b)\tan(x)}{3a^2}$$

input
```
integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
b^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^2) + 1/3*(a*tan(x)
^3 + 3*(a - b)*tan(x))/a^2
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}a^2} + \frac{a^2\tan(x)^3+3a^2\tan(x)-3ab\tan(x)}{3a^3}$$

input
```
integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^2/(sqrt
(a^2 + a*b)*a^2) + 1/3*(a^2*tan(x)^3 + 3*a^2*tan(x) - 3*a*b*tan(x))/a^3
```

3.40.9 Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx = \frac{\tan(x)^3}{3a} - \tan(x)\left(\frac{a+b}{a^2}-\frac{2}{a}\right) + \frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}$$

input
```
int(1/(cos(x)^4*(a + b*cos(x)^2)),x)
```

output
```
tan(x)^3/(3*a) - tan(x)*((a + b)/a^2 - 2/a) + (b^2*atan((a^(1/2)*tan(x))/(
a + b)^(1/2)))/(a^(5/2)*(a + b)^(1/2))
```

3.41 $\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx$

3.41.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx=\frac{b^3\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(a^2-ab+b^2)\tan(x)}{a^3}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a}$$

output
```
b^3*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(7/2)/(a+b)^(1/2)+(a^2-a*b+b^2)*t
an(x)/a^3+1/3*(2*a-b)*tan(x)^3/a^2+1/5*tan(x)^5/a
```

3.41.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx=-\frac{b^3\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(8a^2-10ab+15b^2+a(4a-5b)\sec^2(x)+3a^2\sec^4(x))\tan(x)}{15a^3}$$

input
```
Integrate[Sec[x]^6/(a + b*Cos[x]^2),x]
```

output
```
-((b^3*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b])) + ((8*
a^2 - 10*a*b + 15*b^2 + a*(4*a - 5*b)*Sec[x]^2 + 3*a^2*Sec[x]^4)*Tan[x])/(
15*a^3)
```

3.41.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^6\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$-\int \frac{\left(\cot^2(x)+1\right)^3\tan^6(x)}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{364}$$

$$-\int \left(\frac{\tan^6(x)}{a}+\frac{(2a-b)\tan^4(x)}{a^2}+\frac{\left(a^2-ba+b^2\right)\tan^2(x)}{a^3}+\frac{b^3}{a^3\left(-\left((a+b)\cot^2(x)\right)-a\right)}\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\left(a^2-ab+b^2\right)\tan(x)}{a^3}+\frac{\tan^5(x)}{5a}$$

input
```
Int[Sec[x]^6/(a + b*Cos[x]^2),x]
```

output
```
(b^3*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]) + ((a^2 -
 a*b + b^2)*Tan[x])/a^3 + ((2*a - b)*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a)
```

3.41.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.41.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{\left(\tan^5(x)\right)a^2}{5}+\frac{2a^2\left(\tan^3(x)\right)}{3}-\frac{ab\left(\tan^3(x)\right)}{3}+a^2\tan(x)-ab\tan(x)+b^2\tan(x)}{a^3}-\frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{a^3\sqrt{(a+b)a}}$
risch	$\frac{2i\left(15b^2e^{8ix}-30ab\,e^{6ix}+60b^2e^{6ix}+80a^2e^{4ix}-70\,e^{4ix}ab+90b^2e^{4ix}+40\,e^{2ix}a^2-50b\,e^{2ix}a+60b^2e^{2ix}+8a^2-10ab+15b^2\right)}{15a^3\left(e^{2ix}+1\right)^5}-\frac{b^3\ln\left(e^{2ix}\right.}{}$

input
```
int(sec(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*tan(x)^5*a^2+2/3*a^2*tan(x)^3-1/3*a*b*tan(x)^3+a^2*tan(x)-a*b*t
an(x)+b^2*tan(x))-b^3/a^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.41

$$\int \frac{\sec^6(x)}{a + b\cos^2(x)}\,dx$$

$$= \left[-\frac{15\sqrt{-a^2-ab}b^3\cos(x)^5\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2-4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right.}{60(a^5+a^4}\right.$$

input
```
integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)
^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-
a^2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*((8*a^
4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(x)^4 + 3*a^4 + 3*a^3*b + (4*a^4 -
a^3*b - 5*a^2*b^2)*cos(x)^2)*sin(x))/((a^5 + a^4*b)*cos(x)^5), 1/30*(15*sq
rt(a^2 + a*b)*b^3*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos
(x)*sin(x)))*cos(x)^5 + 2*((8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(x)
^4 + 3*a^4 + 3*a^3*b + (4*a^4 - a^3*b - 5*a^2*b^2)*cos(x)^2)*sin(x))/((a^5
 + a^4*b)*cos(x)^5)]
```

3.41.6 Sympy [F]

$$\int \frac{\sec^6(x)}{a + b\cos^2(x)}\,dx = \int \frac{\sec^6(x)}{a + b\cos^2(x)}\,dx$$

input
```
integrate(sec(x)**6/(a+b*cos(x)**2),x)
```

output
```
Integral(sec(x)**6/(a + b*cos(x)**2), x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx = -\frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^3} + \frac{3\,a^2\tan{(x)}^5 + 5\,(2\,a^2-ab)\tan{(x)}^3 + 15\,(a^2-ab+b^2)\tan{(x)}}{15\,a^3}$$

input
```
integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
-b^3*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^3) + 1/15*(3*a^2*
tan(x)^5 + 5*(2*a^2 - a*b)*tan(x)^3 + 15*(a^2 - a*b + b^2)*tan(x))/a^3
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx = -\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{\sqrt{a^2+ab}a^3} + \frac{3\,a^4\tan{(x)}^5 + 10\,a^4\tan{(x)}^3 - 5\,a^3b\tan{(x)}^3 + 15\,a^4\tan{(x)} - 15\,a^3b\tan{(x)} + 15\,a^2b^2\tan{(x)}}{15\,a^5}$$

input
```
integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
-(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^3/(sqr
t(a^2 + a*b)*a^3) + 1/15*(3*a^4*tan(x)^5 + 10*a^4*tan(x)^3 - 5*a^3*b*tan(x
)^3 + 15*a^4*tan(x) - 15*a^3*b*tan(x) + 15*a^2*b^2*tan(x))/a^5
```

3.41.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx = \frac{\tan(x)^5}{5\,a} - \tan(x)^3\left(\frac{a+b}{3\,a^2}-\frac{1}{a}\right) + \tan(x)\left(\frac{3}{a}+\frac{(a+b)\,\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right) - \frac{b^3\operatorname{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}}$$

input
```
int(1/(cos(x)^6*(a + b*cos(x)^2)),x)
```

output
```
tan(x)^5/(5*a) - tan(x)^3*((a + b)/(3*a^2) - 1/a) + tan(x)*(3/a + ((a + b)
*((a + b)/a^2 - 3/a))/a) - (b^3*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(
7/2)*(a + b)^(1/2))
```

3.41. $\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx$

3.42 $\int \frac{1}{(a+b\cos^2(x))^2}\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx = -\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

output
```
-1/2*(2*a+b)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)-1/2*b*
cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)
```

3.42.2 Mathematica [A] (verified)

Time = 5.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx = -\frac{(-2a-b)\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\sin(2x)}{2a(a+b)(2a+b+b\cos(2x))}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-2),x]
```

output
```
-1/2*((-2*a - b)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*(a + b)^(3
/2)) - (b*Sin[2*x])/(2*a*(a + b)*(2*a + b + b*Cos[2*x]))
```

3.42.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3663, 25, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{2a+b}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a+b}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2a+b)\int \frac{1}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2a+b)\int \frac{1}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{(2a+b)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

input
```
Int[(a + b*Cos[x]^2)^(-2),x]
```

output
```
-1/2*((2*a + b)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(3/
2)) - (b*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos[x]^2))
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.42.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result
default	$-\frac{b\tan(x)}{2(a+b)a(a(\tan^2(x))+a+b)}+\frac{(2a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{2(a+b)a\sqrt{(a+b)a}}$
risch	$-\frac{i(2a\,e^{2ix}+b\,e^{2ix}+b)}{(a+b)a(b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}+b)}-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)}-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a}}{b\sqrt{-a^2-ab}}\right.}{4\sqrt{-a^2-ab}\,(a+b)a}$

input
```
int(1/(a+b*cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(a+b)/a*tan(x)/(a*tan(x)^2+a+b)+1/2*(2*a+b)/(a+b)/a/((a+b)*a)^(1/2)
*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 5.02

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx$$
$$=\left[-\frac{4\left(a^2b+ab^2\right)\cos(x)\sin(x)+\left(\left(2\,ab+b^2\right)\cos(x)^2+2\,a^2+ab\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\,(4\,a}{}\right.}{8\left(a^5+2\,a^4b+a^3b^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)\cos(x)^2\right.}\right.$$
$$\left.-\frac{2\left(a^2b+ab^2\right)\cos(x)\sin(x)+\left(\left(2\,ab+b^2\right)\cos(x)^2+2\,a^2+ab\right)\sqrt{a^2+ab}\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{4\left(a^5+2\,a^4b+a^3b^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)\cos(x)^2\right)}\right]$$

input
```
integrate(1/(a+b*cos(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a^2*b + a*b^2)*cos(x)*sin(x) + ((2*a*b + b^2)*cos(x)^2 + 2*a^2 +
 a*b)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)))/(a^5 + 2*a^4*b + a^3*b^2 +
(a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2), -1/4*(2*(a^2*b + a*b^2)*cos(x)*si
n(x) + ((2*a*b + b^2)*cos(x)^2 + 2*a^2 + a*b)*sqrt(a^2 + a*b)*arctan(1/2*(
(2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^5 + 2*a^4*b +
 a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2)]
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**2)**2,x)
```

output
```
Timed out
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx = -\frac{b\tan(x)}{2\left(a^3+2a^2b+ab^2+(a^3+a^2b)\tan(x)^2\right)} + \frac{(2a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}(a^2+ab)}$$

input
```
integrate(1/(a+b*cos(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b*tan(x)/(a^3 + 2*a^2*b + a*b^2 + (a^3 + a^2*b)*tan(x)^2) + 1/2*(2*a
+ b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + a*b))
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx = \frac{\left(\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(2\,a+b)}{2\,(a^2+ab)^{\frac{3}{2}}} - \frac{b\tan(x)}{2\left(a\tan(x)^2+a+b\right)(a^2+ab)}$$

input
```
integrate(1/(a+b*cos(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(2*a
+ b)/(a^2 + a*b)^(3/2) - 1/2*b*tan(x)/((a*tan(x)^2 + a + b)*(a^2 + a*b))
```

3.42.9 Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,(a+b)^{3/2}} - \frac{b\tan(x)}{2\,a\,(a+b)\,\left(a\tan(x)^2+a+b\right)}$$

input
```
int(1/(a + b*cos(x)^2)^2,x)
```

output
```
(atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*(a + b)^(3/2))
 - (b*tan(x))/(2*a*(a + b)*(a + b + a*tan(x)^2))
```

3.43 $\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx$

3.43.1 Optimal result

Integrand size = 10, antiderivative size = 107

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = -\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}} - \frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}$$

output
```
-1/8*(8*a^2+8*a*b+3*b^2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^
(5/2)-1/4*b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^2-3/8*b*(2*a+b)*cos(x)*si
n(x)/a^2/(a+b)^2/(a+b*cos(x)^2)
```

3.43.2 Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = \frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{\sqrt{a}b(16a^2+16ab+3b^2+3b(2a+b)\cos(2x))\sin(2x)}{(a+b)^2(2a+b+b\cos(2x))^2}}{8a^{5/2}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-3),x]
```

output
```
(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a + b)^(5
/2) - (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 + 3*b*(2*a + b)*Cos[2*x])*Sin[2*
x])/((a + b)^2*(2*a + b + b*Cos[2*x])^2))/(8*a^(5/2))
```

3.43.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos^2(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{-2b\cos^2(x)+4a+3b}{(b\cos^2(x)+a)^2}dx}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2b\cos^2(x)+4a+3b}{(b\cos^2(x)+a)^2}dx}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-2b\sin\left(x+\frac{\pi}{2}\right)^2+4a+3b}{\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^2}dx}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{8a^2+8ba+3b^2}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{-\frac{(8a^2+8ab+3b^2)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

input
```
Int[(a + b*Cos[x]^2)^(-3),x]
```

output
```
-1/4*(b*Cos[x]*Sin[x])/(a*(a + b)*(a + b*Cos[x]^2)^2) + (-1/2*((8*a^2 + 8*
a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(3/2))
 - (3*b*(2*a + b)*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos[x]^2)))/(4*a*(a +
 b))
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.43.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

method	result
default	$\frac{-\frac{b(8a+5b)\left(\tan^3(x)\right)}{8a\left(a^2+2ab+b^2\right)}-\frac{(8a+3b)b\tan(x)}{8a^2(a+b)}}{\left(a(\tan^2(x))+a+b\right)^2}+\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{8(a^2+2ab+b^2)a^2\sqrt{(a+b)a}}$
risch	$-\frac{i\left(8a^2b\,e^{6ix}+8a\,b^2e^{6ix}+3b^3e^{6ix}+48a^3e^{4ix}+72a^2b\,e^{4ix}+42a\,b^2e^{4ix}+9b^3e^{4ix}+40a^2b\,e^{2ix}+40a\,b^2e^{2ix}+9b^3e^{2ix}+6a\,b^2+3b^3\right)}{4(a+b)^2a^2\left(b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}+b\right)^2}-\frac{\ln(}{}$

input
```
int(1/(a+b*cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tan(x)^3-1/8*(8*a+3*b)/a^2*b/(a+b)*tan
(x))/(a*tan(x)^2+a+b)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*b+b^2)/a^2/((a+b)
*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(93) = 186.

Time = 0.29 (sec) , antiderivative size = 616, normalized size of antiderivative = 5.76

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx$$
$$= \left[-\frac{\left((8\,a^2b^2+8\,ab^3+3\,b^4)\cos(x)^4+8\,a^4+8\,a^3b+3\,a^2b^2+2\,(8\,a^3b+8\,a^2b^2+3\,ab^3)\cos(x)^2\right)\sqrt{-a^2-ab}}{32\,\left(a^8+3\,a^7b+3\,a^6b^2+a^5\right.}\right.$$
$$-\frac{\left((8\,a^2b^2+8\,ab^3+3\,b^4)\cos(x)^4+8\,a^4+8\,a^3b+3\,a^2b^2+2\,(8\,a^3b+8\,a^2b^2+3\,ab^3)\cos(x)^2\right)\sqrt{a^2+ab}\,\mathrm{ar}}{16\,\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3+(a^6b^2+3\,a^5b^3+3\,a^4b^4\right.}$$

input
```
integrate(1/(a+b*cos(x)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*
b^2 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(x)^2)*sqrt(-a^2 - a*b)*log(((8
*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*c
os(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*c
os(x)^2 + a^2)) + 4*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 + (8*a^4*b
 + 13*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b + 3*a^6*b^2 + a^
5*b^3 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 + 2*(a^7*b +
3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2), -1/16*(((8*a^2*b^2 + 8*a*b^3 +
 3*b^4)*cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 2*(8*a^3*b + 8*a^2*b^2 +
3*a*b^3)*cos(x)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sq
rt(a^2 + a*b)*cos(x)*sin(x))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x
)^3 + (8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b +
3*a^6*b^2 + a^5*b^3 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4
 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2)]
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**2)**3,x)
```

output
```
Timed out
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \frac{(8\,a^2+8\,ab+3\,b^2)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{8\,(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)a}} - \frac{(8\,a^2b+5\,ab^2)\tan(x)^3+(8\,a^2b+11\,ab^2+3\,b^3)\tan(x)}{8\,(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^6+2\,a^5b+a^4b^2)\tan(x)^4+2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\tan}$$

input
```
integrate(1/(a+b*cos(x)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*arctan(a*tan(x)/sqrt((a + b)*a))/((a^4 + 2*a^3
*b + a^2*b^2)*sqrt((a + b)*a)) - 1/8*((8*a^2*b + 5*a*b^2)*tan(x)^3 + (8*a^
2*b + 11*a*b^2 + 3*b^3)*tan(x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a
^2*b^4 + (a^6 + 2*a^5*b + a^4*b^2)*tan(x)^4 + 2*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*tan(x)^2)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(8\,a^2+8\,ab+3\,b^2)}{8\,(a^4+2\,a^3b+a^2b^2)\sqrt{a^2+ab}} - \frac{8\,a^2b\tan(x)^3+5\,ab^2\tan(x)^3+8\,a^2b\tan(x)+11\,ab^2\tan(x)+3\,b^3\tan(x)}{8\,(a^4+2\,a^3b+a^2b^2)\left(a\tan(x)^2+a+b\right)^2}$$

input
```
integrate(1/(a+b*cos(x)^2)^3,x, algorithm="giac")
```

output
```
1/8*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(8*a^
2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a^2 + a*b)) - 1/8*(8*a^
2*b*tan(x)^3 + 5*a*b^2*tan(x)^3 + 8*a^2*b*tan(x) + 11*a*b^2*tan(x) + 3*b^3
*tan(x))/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(x)^2 + a + b)^2)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,\left(a+b\right)^{5/2}} - \frac{\frac{\tan(x)\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,\left(a+b\right)}+\frac{\tan(x)^3\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,\left(a+b\right)^2}}{2\,a\,b+\tan\left(x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\tan\left(x\right)^4+a^2+b^2}$$

input
```
int(1/(a + b*cos(x)^2)^3,x)
```

output
```
(atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*
(a + b)^(5/2)) - ((tan(x)*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(x)^3*(8*
a*b + 5*b^2))/(8*a*(a + b)^2))/(2*a*b + tan(x)^2*(2*a*b + 2*a^2) + a^2*tan
(x)^4 + a^2 + b^2)
```

3.44 $\int \frac{1}{1+\cos^2(x)}\,dx$

3.44.1 Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \frac{1}{1+\cos^2(x)}\,dx=\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos^2(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

3.44.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3660}$$

$$-\int \frac{1}{2\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{\sqrt{2}}$$

input
```
Int[(1 + Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2])
```

3.44.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.44.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)\sqrt{2}}{2}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{4}$	40

input
```
int(1/(1+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*2^(1/2)*tan(x))*2^(1/2)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+\cos^2(x)}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(x\right)^2-\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

input
```
integrate(1/(1+cos(x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/2 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/2
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.38

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

input
```
int(1/(cos(x)^2 + 1),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/2 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

3.45 $\int \frac{1}{(1+\cos^2(x))^2} \, dx$

3.45.1 Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \frac{1}{(1+\cos^2(x))^2} \, dx = \frac{3x}{4\sqrt{2}} - \frac{3\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}} - \frac{\cos(x)\sin(x)}{4\left(1+\cos^2(x)\right)}$$

output
```
-1/4*cos(x)*sin(x)/(1+cos(x)^2)+3/8*x*2^(1/2)-3/8*arctan(cos(x)*sin(x)/(1+
cos(x)^2+2^(1/2)))*2^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+\cos^2(x))^2} \, dx = \frac{3\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\sin(2x)}{4(3+\cos(2x))}$$

input
```
Integrate[(1 + Cos[x]^2)^(-2),x]
```

output
```
(3*ArcTan[Tan[x]/Sqrt[2]])/(4*Sqrt[2]) - Sin[2*x]/(4*(3 + Cos[2*x]))
```

3.45.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3663, 27, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos^2(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{4}\int -\frac{3}{\cos^2(x)+1}dx-\frac{\sin(x)\cos(x)}{4(\cos^2(x)+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}\int \frac{1}{\cos^2(x)+1}dx-\frac{\sin(x)\cos(x)}{4(\cos^2(x)+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\frac{\sin(x)\cos(x)}{4(\cos^2(x)+1)}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{3}{4}\int \frac{1}{2\cot^2(x)+1}d\cot(x)-\frac{\sin(x)\cos(x)}{4(\cos^2(x)+1)}$$

$$\Big\downarrow \text{216}$$

$$-\frac{3\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}}-\frac{\sin(x)\cos(x)}{4(\cos^2(x)+1)}$$

input
```
Int[(1 + Cos[x]^2)^(-2),x]
```

output
```
(-3*ArcTan[Sqrt[2]*Cot[x]])/(4*Sqrt[2]) - (Cos[x]*Sin[x])/(4*(1 + Cos[x]^2
))
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.45.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.49

method	result	size
default	$-\frac{\tan(x)}{4(\tan^2(x)+2)}+\frac{3\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)\sqrt{2}}{8}$	27
risch	$-\frac{i(3\,\mathrm{e}^{2ix}+1)}{2(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1)}+\frac{3i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{16}-\frac{3i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{16}$	68

input
```
int(1/(1+cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*tan(x)/(tan(x)^2+2)+3/8*arctan(1/2*2^(1/2)*tan(x))*2^(1/2)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(1+\cos^2(x)\right)^2}\,dx = -\frac{3\left(\sqrt{2}\cos\left(x\right)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4\cos\left(x\right)\sin\left(x\right)}{16\left(\cos\left(x\right)^2+1\right)}$$

input
```
integrate(1/(1+cos(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/16*(3*(sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqr
t(2))/(cos(x)*sin(x))) + 4*cos(x)*sin(x))/(cos(x)^2 + 1)
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(53) = 106.

Time = 1.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.96

$$\begin{aligned}\int \frac{1}{\left(1+\cos^2(x)\right)^2}\,dx &= \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+8} \\ &+ \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{8\tan^4\left(\frac{x}{2}\right)+8} \\ &+ \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+8} \\ &+ \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{8\tan^4\left(\frac{x}{2}\right)+8} \\ &+ \frac{2\tan^3\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+8} - \frac{2\tan\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+8}\end{aligned}$$

input
```
integrate(1/(1+cos(x)**2)**2,x)
```

output
```
3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2
)**4/(8*tan(x/2)**4 + 8) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floo
r((x/2 - pi/2)/pi))/(8*tan(x/2)**4 + 8) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2)
 + 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(8*tan(x/2)**4 + 8) + 3*sqr
t(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(8*tan(x/2)*
*4 + 8) + 2*tan(x/2)**3/(8*tan(x/2)**4 + 8) - 2*tan(x/2)/(8*tan(x/2)**4 +
8)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1+\cos^2(x))^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - \frac{\tan(x)}{4\left(\tan(x)^2+2\right)}$$

input
```
integrate(1/(1+cos(x)^2)^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/4*tan(x)/(tan(x)^2 + 2)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+\cos^2(x))^2}\,dx = \frac{3}{8}\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - \sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - \cos(2x) + 1}\right)\right) - \frac{\tan(x)}{4\left(\tan(x)^2+2\right)}$$

input
```
integrate(1/(1+cos(x)^2)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) - 1/4*tan(x)/(tan(x)^2 + 2)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+\cos^2(x))^2}\,dx = \frac{3\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{8} - \frac{\tan(x)}{4\,\left(\tan(x)^2+2\right)} + \frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8}$$

input
```
int(1/(cos(x)^2 + 1)^2,x)
```

output
```
(3*2^(1/2)*(x - atan(tan(x))))/8 - tan(x)/(4*(tan(x)^2 + 2)) + (3*2^(1/2)*
atan((2^(1/2)*tan(x))/2))/8
```

3.46 $\int \frac{1}{(1+\cos^2(x))^3}\,dx$

3.46.1 Optimal result

Integrand size = 8, antiderivative size = 71

$$\int \frac{1}{(1+\cos^2(x))^3}\,dx=\frac{19x}{32\sqrt{2}}-\frac{19\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{32\sqrt{2}}-\frac{\cos(x)\sin(x)}{8\,(1+\cos^2(x))^2}-\frac{9\cos(x)\sin(x)}{32\,(1+\cos^2(x))}$$

output
```
-1/8*cos(x)*sin(x)/(1+cos(x)^2)^2-9/32*cos(x)*sin(x)/(1+cos(x)^2)+19/64*x*
2^(1/2)-19/64*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1+\cos^2(x))^3}\,dx=\frac{19\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{\sin(2x)}{4(3+\cos(2x))^2}-\frac{9\sin(2x)}{32(3+\cos(2x))}$$

input
```
Integrate[(1 + Cos[x]^2)^(-3),x]
```

output
```
(19*ArcTan[Tan[x]/Sqrt[2]])/(32*Sqrt[2]) - Sin[2*x]/(4*(3 + Cos[2*x])^2) -
 (9*Sin[2*x])/(32*(3 + Cos[2*x]))
```

3.46.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cos^2(x)+1\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{8}\int -\frac{7-2\cos^2(x)}{\left(\cos^2(x)+1\right)^2}dx-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8}\int \frac{7-2\cos^2(x)}{\left(\cos^2(x)+1\right)^2}dx-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \frac{7-2\sin\left(x+\frac{\pi}{2}\right)^2}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^2}dx-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{8}\left(\frac{1}{4}\int \frac{19}{\cos^2(x)+1}dx-\frac{9\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)}\right)-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{19}{4}\int \frac{1}{\cos^2(x)+1}dx-\frac{9\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)}\right)-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{19}{4}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\frac{9\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)}\right)-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{8}\left(-\frac{19}{4}\int \frac{1}{2\cot^2(x)+1}d\cot(x)-\frac{9\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)}\right)-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

$$\Big\downarrow 216$$

$$\frac{1}{8}\left(-\frac{19\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}}-\frac{9\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)}\right)-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

input
```
Int[(1 + Cos[x]^2)^(-3),x]
```

output
```
-1/8*(Cos[x]*Sin[x])/(1 + Cos[x]^2)^2 + ((-19*ArcTan[Sqrt[2]*Cot[x]])/(4*S
qrt[2]) - (9*Cos[x]*Sin[x])/(4*(1 + Cos[x]^2)))/8
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.46.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{-\frac{13\left(\tan^3(x)\right)}{32}-\frac{11\tan(x)}{16}}{\left(\tan^2(x)+2\right)^2}+\frac{19\arctan\left(\frac{\sqrt{2}\tan(x)}{2}\right)\sqrt{2}}{64}$	35
risch	$-\frac{i(19\,\mathrm{e}^{6ix}+171\,\mathrm{e}^{4ix}+89\,\mathrm{e}^{2ix}+9)}{16(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1)^2}+\frac{19i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{128}-\frac{19i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{128}$	82

input
```
int(1/(1+cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-13/32*tan(x)^3-11/16*tan(x))/(tan(x)^2+2)^2+19/64*arctan(1/2*2^(1/2)*tan
(x))*2^(1/2)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(1+\cos^2(x)\right)^3}\,dx = -\frac{19\left(\sqrt{2}\cos(x)^4+2\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4\left(9\cos(x)^3+13\cos(x)\right)\sin(x)}{128\left(\cos(x)^4+2\cos(x)^2+1\right)}$$

input
```
integrate(1/(1+cos(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/128*(19*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3
*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) + 4*(9*cos(x)^3 + 13*cos(x))
*sin(x))/(cos(x)^4 + 2*cos(x)^2 + 1)
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(71) = 142$.

Time = 4.22 (sec) , antiderivative size = 439, normalized size of antiderivative = 6.18

$$\int \frac{1}{(1+\cos^2(x))^3}\,dx = \frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^8\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{38\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^8\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{38\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{22\tan^7\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} - \frac{14\tan^5\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} + \frac{14\tan^3\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64} - \frac{22\tan\left(\frac{x}{2}\right)}{64\tan^8\left(\frac{x}{2}\right)+128\tan^4\left(\frac{x}{2}\right)+64}$$

input
```
integrate(1/(1+cos(x)**2)**3,x)
```

output
```
19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/
2)**8/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 38*sqrt(2)*(atan(sqrt(2)*t
an(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(64*tan(x/2)**8 + 12
8*tan(x/2)**4 + 64) + 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x
/2 - pi/2)/pi))/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 19*sqrt(2)*(atan
(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**8/(64*tan(x/
2)**8 + 128*tan(x/2)**4 + 64) + 38*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + p
i*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(64*tan(x/2)**8 + 128*tan(x/2)**4 +
64) + 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/
(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 22*tan(x/2)**7/(64*tan(x/2)**8 +
 128*tan(x/2)**4 + 64) - 14*tan(x/2)**5/(64*tan(x/2)**8 + 128*tan(x/2)**4
+ 64) + 14*tan(x/2)**3/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) - 22*tan(x/
2)/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1+\cos^2(x))^3}\,dx = \frac{19}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - \frac{13\tan(x)^3 + 22\tan(x)}{32\left(\tan(x)^4 + 4\tan(x)^2 + 4\right)}$$

input
```
integrate(1/(1+cos(x)^2)^3,x, algorithm="maxima")
```

output
```
19/64*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/32*(13*tan(x)^3 + 22*tan(x))/
(tan(x)^4 + 4*tan(x)^2 + 4)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1+\cos^2(x))^3}\,dx = \frac{19}{64}\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - \sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - \cos(2x) + 1}\right)\right) - \frac{13\tan(x)^3 + 22\tan(x)}{32\left(\tan(x)^2 + 2\right)^2}$$

input
```
integrate(1/(1+cos(x)^2)^3,x, algorithm="giac")
```

output
```
19/64*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - cos(2*x) + 1))) - 1/32*(13*tan(x)^3 + 22*tan(x))/(tan(x)^2 +
2)^2
```

3.46.9 Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+\cos^2(x))^3}\,dx = \frac{19\sqrt{2}\,(x-\mathrm{atan}(\tan(x)))}{64} - \frac{\frac{13\tan(x)^3}{32}+\frac{11\tan(x)}{16}}{\tan(x)^4+4\tan(x)^2+4} + \frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{64}$$

input
```
int(1/(cos(x)^2 + 1)^3,x)
```

output
```
(19*2^(1/2)*(x - atan(tan(x))))/64 - ((11*tan(x))/16 + (13*tan(x)^3)/32)/(
4*tan(x)^2 + tan(x)^4 + 4) + (19*2^(1/2)*atan((2^(1/2)*tan(x))/2))/64
```

3.47 $\int \sqrt{1-\cos^2(x)}\,dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \sqrt{1-\cos^2(x)}\,dx = -\cot(x)\sqrt{\sin^2(x)}$$

output
```
-cot(x)*(sin(x)^2)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\cos^2(x)}\,dx = -\cot(x)\sqrt{\sin^2(x)}$$

input
```
Integrate[Sqrt[1 - Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[Sin[x]^2])
```

3.47.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3655, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sqrt{\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx$$

$$\Big\downarrow \text{3118}$$

$$\sqrt{\sin^2(x)}(-\cot(x))$$

input
```
Int[Sqrt[1 - Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[Sin[x]^2])
```

3.47.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.47.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\sin(x)\cos(x)}{\sqrt{2-2\cos(2x)}}$	13
risch	$-\frac{i\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}\, e^{2ix}}{2(e^{2ix}-1)}-\frac{i\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}{2(e^{2ix}-1)}$	67

input
```
int((1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)*cos(x)/(sin(x)^2)^(1/2)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.33

$$\int \sqrt{1-\cos^2(x)}\,dx = -\cos(x)$$

input
```
integrate((1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-cos(x)
```

3.47.6 Sympy [F]

$$\int \sqrt{1-\cos^2(x)}\,dx = \int \sqrt{1-\cos^2(x)}\,dx$$

input
```
integrate((1-cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - cos(x)**2), x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1-\cos^2(x)}\,dx = -\frac{1}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate((1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/sqrt(tan(x)^2 + 1)
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \sqrt{1-\cos^2(x)}\,dx = -\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}$$

input
```
integrate((1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sgn(tan(1/2*x)^3 + tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1-\cos^2(x)}\,dx = -\cot(x)\,\sqrt{\sin\left(x\right)^2}$$

input
```
int((1 - cos(x)^2)^(1/2),x)
```

output
```
-cot(x)*(sin(x)^2)^(1/2)
```

3.48 $\int \sqrt{-1+\cos^2(x)}\,dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{-1+\cos^2(x)}\,dx=-\cot(x)\sqrt{-\sin^2(x)}$$

output
```
-cot(x)*(-sin(x)^2)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+\cos^2(x)}\,dx=-\cot(x)\sqrt{-\sin^2(x)}$$

input
```
Integrate[Sqrt[-1 + Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[-Sin[x]^2])
```

3.48.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3655, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos^2(x)-1}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2-1}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \sqrt{-\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{-\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\sqrt{-\sin^2(x)}\csc(x)\int \sin(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{-\sin^2(x)}\csc(x)\int \sin(x)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\sqrt{-\sin^2(x)}(-\cot(x))$$

input
```
Int[Sqrt[-1 + Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[-Sin[x]^2])
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.48.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sin(x)\cos(x)}{\sqrt{-\left(\sin^2(x)\right)}}$	14
risch	$-\frac{i\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}\, e^{2ix}}{2\left(e^{2ix}-1\right)}-\frac{i\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}{2\left(e^{2ix}-1\right)}$	65

input
```
int((-1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin(x)*cos(x)/(-sin(x)^2)^(1/2)
```

3.48.5 Fricas [F]

$$\int \sqrt{-1+\cos^2(x)}\,dx = \int \sqrt{\cos(x)^2 - 1}\,dx$$

input
```
integrate((-1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
0
```

3.48.6 Sympy [F]

$$\int \sqrt{-1+\cos^2(x)}\,dx = \int \sqrt{\cos^2(x) - 1}\,dx$$

input
```
integrate((-1+cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cos(x)**2 - 1), x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{-1+\cos^2(x)}\,dx = -\frac{1}{\sqrt{-\tan(x)^2 - 1}}$$

input
```
integrate((-1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/sqrt(-tan(x)^2 - 1)
```

3.48.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \sqrt{-1+\cos^2(x)}\,dx = \frac{2i\,\mathrm{sgn}\Big(-\tan\left(\frac{1}{2}\,x\right)^3 - \tan\left(\frac{1}{2}\,x\right)\Big)}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

input
```
integrate((-1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*I*sgn(-tan(1/2*x)^3 - tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \sqrt{-1+\cos^2(x)}\,dx = -\frac{\sqrt{-4\sin(x)^2}\left(-\sin(x)^2 + \frac{\sin(2\,x)\,1\mathrm{i}}{2} + 1\right)}{\sin(x)^2\,2\mathrm{i} + \sin(2\,x)}$$

input
```
int((cos(x)^2 - 1)^(1/2),x)
```

output
```
-((-4*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(sin(2*x) + sin(x)
^2*2i)
```

3.49 $\int \left(1-\cos^2(x)\right)^{3/2}\,dx$

3.49.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \left(1-\cos^2(x)\right)^{3/2}\,dx=-\frac{2}{3}\cot(x)\sqrt{\sin^2(x)}-\frac{1}{3}\cot(x)\sin^2(x)^{3/2}$$

output
```
-1/3*cot(x)*(sin(x)^2)^(3/2)-2/3*cot(x)*(sin(x)^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \left(1-\cos^2(x)\right)^{3/2}\,dx=\frac{1}{12}(-9\cos(x)+\cos(3x))\csc(x)\sqrt{\sin^2(x)}$$

input
```
Integrate[(1 - Cos[x]^2)^(3/2),x]
```

output
```
((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[Sin[x]^2])/12
```

3.49.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\cos^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sin^2(x)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sin(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}\int \sqrt{\sin^2(x)}dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \sqrt{\sin(x)^2}dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)-\frac{2}{3}\sqrt{\sin^2(x)}\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(3/2),x]
```

output
```
(-2*Cot[x]*Sqrt[Sin[x]^2])/3 - (Cot[x]*(Sin[x]^2)^(3/2))/3
```

3.49.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.49.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2\sin(x)\cos(x)(\cos^2(x)-3)}{3\sqrt{2-2\cos(2x)}}$	19
risch	$\frac{i e^{4ix}\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}{24\,e^{2ix}-24}-\frac{3i\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}\,e^{2ix}}{8(e^{2ix}-1)}-\frac{3i\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}{8(e^{2ix}-1)}+\frac{i e^{-2ix}\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}{24\,e^{2ix}-24}$	137

input
```
int((1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(x)*cos(x)*(cos(x)^2-3)/(sin(x)^2)^(1/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.38

$$\int \left(1-\cos^2(x)\right)^{3/2}\,dx=\frac{1}{3}\cos(x)^3-\cos(x)$$

input
```
integrate((1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \left(1-\cos^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((1-cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.38

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = -\frac{1}{12}\cos(3x) + \frac{3}{4}\cos(x)$$

input
```
integrate((1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*cos(3*x) + 3/4*cos(x)
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = -\frac{4\left(3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2+\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

input
```
integrate((1-cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-4/3*(3*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2 + sgn(tan(1/2*x)^3 + t
an(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = \int \left(1-\cos(x)^2\right)^{3/2} dx$$

input
```
int((1 - cos(x)^2)^(3/2),x)
```

output
```
int((1 - cos(x)^2)^(3/2), x)
```

3.50 $\int \left(-1+\cos^2(x)\right)^{3/2}\,dx$

3.50.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx=\frac{2}{3}\cot(x)\sqrt{-\sin^2(x)}-\frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2}$$

output
```
-1/3*cot(x)*(-sin(x)^2)^(3/2)+2/3*cot(x)*(-sin(x)^2)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx=-\frac{1}{12}(-9\cos(x)+\cos(3x))\csc(x)\sqrt{-\sin^2(x)}$$

input
```
Integrate[(-1 + Cos[x]^2)^(3/2),x]
```

output
```
-1/12*((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[-Sin[x]^2])
```

3.50.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cos^2(x)-1\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sin\left(x+\frac{\pi}{2}\right)^2-1\right)^{3/2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(-\sin^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$-\frac{2}{3}\int \sqrt{-\sin^2(x)}dx-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\int \sqrt{-\sin(x)^2}dx-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3686}$$

$$-\frac{2}{3}\sqrt{-\sin^2(x)}\csc(x)\int \sin(x)dx-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\sqrt{-\sin^2(x)}\csc(x)\int \sin(x)dx-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3118}$$

$$\frac{2}{3}\sqrt{-\sin^2(x)}\cot(x)-\frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

input
```
Int[(-1 + Cos[x]^2)^(3/2),x]
```

output
```
(2*Cot[x]*Sqrt[-Sin[x]^2])/3 - (Cot[x]*(-Sin[x]^2)^(3/2))/3
```

3.50.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.50.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\sqrt{-\left(\sin^{2}\left(x\right)\right)}\, \cos\left(x\right) \left(\cos^{2}\left(x\right)-3\right)}{3 \sin\left(x\right)}$	23
risch	$-\frac{i {\mathrm e}^{4 i x} \sqrt{\left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \left({\mathrm e}^{2 i x}-1\right)}+\frac{3 i \sqrt{\left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}\, {\mathrm e}^{2 i x}}{8 \left({\mathrm e}^{2 i x}-1\right)}+\frac{3 i \sqrt{\left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}-1\right)}-\frac{i {\mathrm e}^{-2 i x} \sqrt{\left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \left({\mathrm e}^{2 i x}-1\right)}$	133

input
```
int((-1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-sin(x)^2)^(1/2)*cos(x)*(cos(x)^2-3)/sin(x)
```

3.50.5 Fricas [F]

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx=\int \left(\cos\left(x\right)^{2}-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
0
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-1+cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos\left(x\right)^2-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 - 1)^(3/2), x)
```

3.50.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx = -\frac{4\left(3i\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2+i\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

input
```
integrate((-1+cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-4/3*(3*I*sgn(-tan(1/2*x)^3 - tan(1/2*x))*tan(1/2*x)^2 + I*sgn(-tan(1/2*x)
^3 - tan(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos\left(x\right)^2-1\right)^{3/2}\,dx$$

input
```
int((cos(x)^2 - 1)^(3/2),x)
```

output
```
int((cos(x)^2 - 1)^(3/2), x)
```

3.51 $\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$

3.51.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{\sin^2(x)}}$$

output
```
-arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{\sin^2(x)}}$$

input
```
Integrate[1/Sqrt[1 - Cos[x]^2],x]
```

output
```
((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[Sin[x]^2]
```

3.51.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3655, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sqrt{\sin^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin(x)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{\sqrt{\sin^2(x)}}$$

input
```
Int[1/Sqrt[1 - Cos[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[Sin[x]^2])
```

3.51.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.51.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{2\,\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{2-2\cos(2x)}}$	14
risch	$\frac{2\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\,\mathrm{e}^{-2ix}}}-\frac{2\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\,\mathrm{e}^{-2ix}}}$	62

input
```
int(1/(1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = -\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

3.51.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$$

input
```
integrate(1/(1-cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - cos(x)**2), x)
```

3.51.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) - \frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)}$$

input
```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*x)))/sgn(tan(1/2*x)^3 + tan(1/2*x))
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cos\left(x\right)^2}}\,dx$$

input
```
int(1/(1 - cos(x)^2)^(1/2),x)
```

output
```
int(1/(1 - cos(x)^2)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx=-\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{-\sin^2(x)}}$$

output
```
-arctanh(cos(x))*sin(x)/(-sin(x)^2)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx=\frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{-\sin^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 + Cos[x]^2],x]
```

output
```
((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[-Sin[x]^2]
```

3.52.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3655, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos^2(x)-1}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx$$
$$\Big\downarrow \text{3655}$$
$$\int \frac{1}{\sqrt{-\sin^2(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{-\sin(x)^2}}dx$$
$$\Big\downarrow \text{3686}$$
$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{-\sin^2(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{-\sin^2(x)}}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{\sqrt{-\sin^2(x)}}$$

input
```
Int[1/Sqrt[-1 + Cos[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[-Sin[x]^2])
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

method	result	size
default	$-\frac{\sin(x)\sqrt{-(\cos^2(x))}\,\arctan\left(\frac{1}{\sqrt{-(\cos^2(x))}}\right)}{\cos(x)\sqrt{-(\sin^2(x))}}$	34
risch	$\frac{2\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}-\frac{2\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}$	60

input
```
int(1/(-1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)*(-cos(x)^2)^(1/2)*arctan(1/(-cos(x)^2)^(1/2))/cos(x)/(-sin(x)^2)^(
1/2)
```

3.52.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.52.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos^2{(x)}-1}}\,dx$$

input
```
integrate(1/(-1+cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(cos(x)**2 - 1), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx = -\arctan(\sin{(x)},\cos{(x)}+1) + \arctan(\sin{(x)},\cos{(x)}-1)$$

input
```
integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arctan2(sin(x), cos(x) + 1) + arctan2(sin(x), cos(x) - 1)
```

3.52.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx=\frac{i\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)\right)}$$

input
```
integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
I*log(abs(tan(1/2*x)))/sgn(-tan(1/2*x)^3 - tan(1/2*x))
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx=\int \frac{1}{\sqrt{\cos\left(x\right)^2-1}}\,dx$$

input
```
int(1/(cos(x)^2 - 1)^(1/2),x)
```

output
```
int(1/(cos(x)^2 - 1)^(1/2), x)
```

3.53 $\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}} \, dx$

3.53.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}} \, dx = -\frac{\cot(x)}{2\sqrt{\sin^2(x)}} - \frac{\operatorname{arctanh}(\cos(x))\sin(x)}{2\sqrt{\sin^2(x)}}$$

output
```
-1/2*cot(x)/(sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}} \, dx = -\frac{\left(\csc^2\left(\frac{x}{2}\right) + 4\log\left(\cos\left(\frac{x}{2}\right)\right) - 4\log\left(\sin\left(\frac{x}{2}\right)\right) - \sec^2\left(\frac{x}{2}\right)\right)\sin(x)}{8\sqrt{\sin^2(x)}}$$

input
```
Integrate[(1 - Cos[x]^2)^(-3/2),x]
```

output
```
-1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]
)/Sqrt[Sin[x]^2]
```

3.53.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3655, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \frac{1}{\sin^2(x)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(\sin(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3683}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin^2(x)}}dx-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(x)^2}}dx-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

input
```
Int[(1 - Cos[x]^2)^(-3/2),x]
```

output
```
-1/2*Cot[x]/Sqrt[Sin[x]^2] - (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[Sin[x]^2])
```

3.53.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.53.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{2\left(\frac{\cos(x)}{2}+\frac{(-\ln(\cos(x)-1)+\ln(1+\cos(x)))\left(\sin^2(x)\right)}{4}\right)}{\sin(x)\sqrt{2-2\cos(2x)}}$	37
risch	$-\frac{i(e^{2ix}+1)}{(e^{2ix}-1)\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}+\frac{\ln(e^{ix}-1)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}-\frac{\ln(e^{ix}+1)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}$	98

input
```
int(1/(1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(1/2*cos(x)+1/4*(-ln(cos(x)-1)+ln(1+cos(x)))*sin(x)^2)/sin(x)/(sin(x)^2)^
(1/2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = -\frac{(\cos(x)^2-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)^2-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4(\cos(x)^2-1)}$$

input
```
integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x
) + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

3.53.6 Sympy [F]

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(1-\cos^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1-cos(x)**2)**(3/2),x)
```

output
```
Integral((1 - cos(x)**2)**(-3/2), x)
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(24) = 48$.

Time = 0.48 (sec) , antiderivative size = 300, normalized size of antiderivative = 9.38

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = \frac{4\left(\cos{(3x)}+\cos{(x)}\right)\cos{(4x)}-4\left(2\cos{(2x)}-1\right)\cos{(3x)}-8\cos{(2x)}\cos{(x)}+}{}$$

input
```
integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - s
in(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x
)^2 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2
 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(
2*x) - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*
sin(4*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(2
*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.44

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = \frac{\tan\left(\frac{1}{2}\,x\right)^2}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)} + \frac{\log\left(\tan\left(\frac{1}{2}\,x\right)^2\right)}{4\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)} - \frac{2\,\tan\left(\frac{1}{2}\,x\right)^2+1}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2}$$

input
```
integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*tan(1/2*x)^2/sgn(tan(1/2*x)^3 + tan(1/2*x)) + 1/4*log(tan(1/2*x)^2)/sg
n(tan(1/2*x)^3 + tan(1/2*x)) - 1/8*(2*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x)^3
+ tan(1/2*x))*tan(1/2*x)^2)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(1-\cos\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(1 - cos(x)^2)^(3/2),x)
```

output
```
int(1/(1 - cos(x)^2)^(3/2), x)
```

3.54 $\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx$

3.54.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx=\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}+\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{2\sqrt{-\sin^2(x)}}$$

output
```
1/2*cot(x)/(-sin(x)^2)^(1/2)+1/2*arctanh(cos(x))*sin(x)/(-sin(x)^2)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx=\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin(x)}{8\sqrt{-\sin^2(x)}}$$

input
```
Integrate[(-1 + Cos[x]^2)^(-3/2),x]
```

output
```
((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x])/(8*
Sqrt[-Sin[x]^2])
```

3.54.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3655, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos^2(x)-1)^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2-1\right)^{3/2}}dx$$

$$\Big\downarrow \; 3655$$

$$\int \frac{1}{\left(-\sin^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(-\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \; 3683$$

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}-\frac{1}{2}\int \frac{1}{\sqrt{-\sin^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}-\frac{1}{2}\int \frac{1}{\sqrt{-\sin(x)^2}}dx$$

$$\Big\downarrow \; 3686$$

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}-\frac{\sin(x)\int \csc(x)dx}{2\sqrt{-\sin^2(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}-\frac{\sin(x)\int \csc(x)dx}{2\sqrt{-\sin^2(x)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{\sin(x)\text{arctanh}(\cos(x))}{2\sqrt{-\sin^2(x)}}+\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}$$

input
```
Int[(-1 + Cos[x]^2)^(-3/2),x]
```

output
```
Cot[x]/(2*Sqrt[-Sin[x]^2]) + (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[-Sin[x]^2])
```

3.54.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.54.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{-\frac{\cos(x)}{2}+\frac{(\ln(\cos(x)-1)-\ln(1+\cos(x)))\left(\sin^2(x)\right)}{4}}{\sin(x)\sqrt{-\left(\sin^2(x)\right)}}$	39
risch	$\frac{i(\mathrm{e}^{2ix}+1)}{(\mathrm{e}^{2ix}-1)\sqrt{(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}+\frac{\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}-\frac{\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}$	95

input
```
int(1/(-1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1/2*cos(x)+1/4*(ln(cos(x)-1)-ln(1+cos(x)))*sin(x)^2)/sin(x)/(-sin(x)^2)
^(1/2)
```

3.54.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(-1+\cos^2(x))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.54.6 Sympy [F]

$$\int \frac{1}{(-1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos^2(x)-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-1+cos(x)**2)**(3/2),x)
```

output
```
Integral((cos(x)**2 - 1)**(-3/2), x)
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 284, normalized size of antiderivative = 7.89

$$\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx = \frac{\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\mathrm{s}\right.}{}$$

input
```
integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), co
s(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin
(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin
(x), cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*
sin(4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*si
n(2*x) - 4*cos(2*x)*sin(x) + 2*sin(x))/(2*(2*cos(2*x) - 1)*cos(4*x) - cos(
4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 +
4*cos(2*x) - 1)
```

3.54.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\begin{aligned}\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx &= -\frac{i\tan\left(\frac{1}{2}x\right)^2}{8\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)} \\ &\quad -\frac{i\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)}+\frac{2i\tan\left(\frac{1}{2}x\right)^2+i}{8\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2}\end{aligned}$$

input
```
integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*I*tan(1/2*x)^2/sgn(-tan(1/2*x)^3 - tan(1/2*x)) - 1/4*I*log(tan(1/2*x)
^2)/sgn(-tan(1/2*x)^3 - tan(1/2*x)) + 1/8*(2*I*tan(1/2*x)^2 + I)/(sgn(-tan
(1/2*x)^3 - tan(1/2*x))*tan(1/2*x)^2)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(\cos\left(x\right)^2-1\right)^{3/2}}\,dx$$

input
```
int(1/(cos(x)^2 - 1)^(3/2),x)
```

output
```
int(1/(cos(x)^2 - 1)^(3/2), x)
```

3.55 $\int \sqrt{1+\cos^2(x)}\,dx$

3.55.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \sqrt{1+\cos^2(x)}\,dx = E\left(\frac{\pi}{2}+x\middle|-1\right)$$

output
```
-(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)
```

3.55.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \sqrt{1+\cos^2(x)}\,dx = \sqrt{2}E\left(x\middle|\frac{1}{2}\right)$$

input
```
Integrate[Sqrt[1 + Cos[x]^2],x]
```

output
```
Sqrt[2]*EllipticE[x, 1/2]
```

3.55.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3656}$$

$$E\left(x+\frac{\pi}{2}\middle|-1\right)$$

input
```
Int[Sqrt[1 + Cos[x]^2],x]
```

output
```
EllipticE[Pi/2 + x, -1]
```

3.55.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

3.55.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.

Time = 2.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.56

method	result	size
default	$-\frac{\sqrt{\left(1+\cos^{2}\left(x\right)\right)\left(\sin^{2}\left(x\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos\left(2x\right)}{2}}\,E\left(\cos\left(x\right),i\right)}{\sqrt{1-\left(\cos^{4}\left(x\right)\right)}\,\sin\left(x\right)}$	41

input
```
int((1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)*EllipticE(cos(x),I)/(1-cos
(x)^4)^(1/2)/sin(x)
```

3.55.5 Fricas [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos(x)^2+1}\,dx$$

input
```
integrate((1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(x)^2 + 1), x)
```

3.55.6 Sympy [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos^2(x)+1}\,dx$$

input
```
integrate((1+cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cos(x)**2 + 1), x)
```

3.55.7 Maxima [F]

$$\int \sqrt{1+\cos^2(x)}\,dx=\int \sqrt{\cos\left(x\right)^2+1}\,dx$$

input
```
integrate((1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(x)^2 + 1), x)
```

3.55.8 Giac [F]

$$\int \sqrt{1+\cos^2(x)}\,dx=\int \sqrt{\cos\left(x\right)^2+1}\,dx$$

input
```
integrate((1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(x)^2 + 1), x)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sqrt{1+\cos^2(x)}\,dx=\sqrt{2}\,\mathrm{E}\left(x\middle|\frac{1}{2}\right)$$

input
```
int((cos(x)^2 + 1)^(1/2),x)
```

output
```
2^(1/2)*ellipticE(x, 1/2)
```

3.56 $\int \sqrt{-1-\cos^2(x)}\,dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{-1-\cos^2(x)}\,dx = \frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}$$

output
```
-(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)
^2)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \sqrt{-1-\cos^2(x)}\,dx = -\frac{\sqrt{2}\sqrt{3+\cos(2x)}\,E\left(x\middle|\frac{1}{2}\right)}{\sqrt{-3-\cos(2x)}}$$

input
```
Integrate[Sqrt[-1 - Cos[x]^2],x]
```

output
```
-((Sqrt[2]*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2])/Sqrt[-3 - Cos[2*x]])
```

3.56.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\cos^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{-\cos^2(x)-1}\int \sqrt{\cos^2(x)+1}dx}{\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos^2(x)-1}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{-\cos^2(x)-1}E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

input
```
Int[Sqrt[-1 - Cos[x]^2],x]
```

output
```
(Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2]
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.56.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(35) = 70.

Time = 2.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

method	result	size
default	$-\frac{i\sqrt{-(1+\cos^2(x))(\sin^2(x))}\,\sqrt{1+\cos^2(x)}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,(2F(i\cos(x),i)-E(i\cos(x),i))}{\sqrt{-1+\cos^4(x)}\,\sin(x)\sqrt{-1-(\cos^2(x))}}$	75

input
```
int((-1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*(sin(x)^2)^(1/2)*(2*E
llipticF(I*cos(x),I)-EllipticE(I*cos(x),I))/(-1+cos(x)^4)^(1/2)/sin(x)/(-1
-cos(x)^2)^(1/2)
```

3.56.5 Fricas [F]

$$\int \sqrt{-1-\cos^2(x)}\,dx = \int \sqrt{-\cos(x)^2-1}\,dx$$

input
```
integrate((-1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*(e^(2*I*x) - e^(I*x))*integral(4*sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*
(e^(2*I*x) + 1)/(e^(6*I*x) - 2*e^(5*I*x) + 7*e^(4*I*x) - 12*e^(3*I*x) + 7*
e^(2*I*x) - 2*e^(I*x) + 1), x) + sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(e^(I*x
) + 1))/(e^(2*I*x) - e^(I*x))
```

3.56.6 Sympy [F]

$$\int \sqrt{-1-\cos^2(x)}\,dx = \int \sqrt{-\cos^2(x)-1}\,dx$$

input
```
integrate((-1-cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-cos(x)**2 - 1), x)
```

3.56.7 Maxima [F]

$$\int \sqrt{-1-\cos^2(x)}\,dx = \int \sqrt{-\cos(x)^2-1}\,dx$$

input
```
integrate((-1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-cos(x)^2 - 1), x)
```

3.56.8 Giac [F]

$$\int \sqrt{-1-\cos^2(x)}\,dx = \int \sqrt{-\cos(x)^2 - 1}\,dx$$

input
```
integrate((-1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cos(x)^2 - 1), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-1-\cos^2(x)}\,dx = \int \sqrt{-\cos(x)^2 - 1}\,dx$$

input
```
int((- cos(x)^2 - 1)^(1/2),x)
```

output
```
int((- cos(x)^2 - 1)^(1/2), x)
```

3.57 $\int \sqrt{a+b\cos^2(x)}\,dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \sqrt{a+b\cos^2(x)}\,dx = \frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{\sqrt{1+\frac{b\cos^2(x)}{a}}}$$

output
```
-(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/
2)/(1+b*cos(x)^2/a)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \sqrt{a+b\cos^2(x)}\,dx = \frac{\sqrt{2a+b+b\cos(2x)}\,E\left(x\middle|\frac{b}{a+b}\right)}{\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[2*a + b + b*Cos[2*x]]*EllipticE[x, b/(a + b)])/Sqrt[(2*a + b + b*Cos
[2*x])/(a + b)]
```

3.57.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (b*Cos[x]^2)/a
]
```

3.57.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.57.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^2(x)\right)}{a}}\,E\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b(\cos^2(x))}}$	49

input
```
int((a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2)
)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

3.57.5 Fricas [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{b\cos\left(x\right)^2+a}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(x)^2 + a), x)
```

3.57.6 Sympy [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{a+b\cos^2{(x)}}\,dx$$

input
```
integrate((a+b*cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(x)**2), x)
```

3.57.7 Maxima [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{b\cos{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(x)^2 + a), x)
```

3.57.8 Giac [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{b\cos{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(x)^2 + a), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{b\cos{(x)}^2+a}\,dx$$

input
```
int((a + b*cos(x)^2)^(1/2),x)
```

output
```
int((a + b*cos(x)^2)^(1/2), x)
```

3.58 $\int \left(1+\cos^2(x)\right)^{3/2} dx$

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \left(1+\cos^2(x)\right)^{3/2} dx = 2E\left(\frac{\pi}{2}+x\middle|-1\right) - \frac{2}{3}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right) + \frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\sin(x)$$

output
```
-2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)+2/3*(sin(x)^2)^(1/2)/sin(x)
*EllipticF(cos(x),I)+1/3*cos(x)*sin(x)*(1+cos(x)^2)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(1+\cos^2(x)\right)^{3/2} dx = \frac{24E\left(x\middle|\frac{1}{2}\right) - 4\,\text{EllipticF}\left(x,\frac{1}{2}\right) + \sqrt{3+\cos(2x)}\sin(2x)}{6\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(3/2),x]
```

output
```
(24*EllipticE[x, 1/2] - 4*EllipticF[x, 1/2] + Sqrt[3 + Cos[2*x]]*Sin[2*x])
/(6*Sqrt[2])
```

3.58.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cos^2(x)+1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3659}$$

$$\frac{1}{3}\int \frac{2\left(3\cos^2(x)+2\right)}{\sqrt{\cos^2(x)+1}}dx+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{3\cos^2(x)+2}{\sqrt{\cos^2(x)+1}}dx+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\int \frac{3\sin\left(x+\frac{\pi}{2}\right)^2+2}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3651}$$

$$\frac{2}{3}\left(3\int \sqrt{\cos^2(x)+1}dx-\int \frac{1}{\sqrt{\cos^2(x)+1}}dx\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\left(3\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3656}$$

$$\frac{2}{3}\left(3E\left(x+\frac{\pi}{2}\middle|-1\right)-\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3661}$$

$$\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}+\frac{2}{3}\left(3E\left(x+\frac{\pi}{2}\middle|-1\right)-\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)\right)$$

input
```
Int[(1 + Cos[x]^2)^(3/2),x]
```

output
```
(2*(3*EllipticE[Pi/2 + x, -1] - EllipticF[Pi/2 + x, -1]))/3 + (Cos[x]*Sqrt
[1 + Cos[x]^2]*Sin[x])/3
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

3.58.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.

Time = 1.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.35

method	result
default	$\frac{\sqrt{(1+\cos^2(x))(\sin^2(x))}\left(-(\sin^4(x))\cos(x)+2\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{2-(\sin^2(x))}\,F(\cos(x),i)-6\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{2-(\sin^2(x))}\,E(\cos(x),i)+\right.}{3\sqrt{1-(\cos^4(x))}\sin(x)\sqrt{1+\cos^2(x)}}$

input
```
int((1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((1+cos(x)^2)*sin(x)^2)^(1/2)*(-sin(x)^4*cos(x)+2*(sin(x)^2)^(1/2)*(2-
sin(x)^2)^(1/2)*EllipticF(cos(x),I)-6*(sin(x)^2)^(1/2)*(2-sin(x)^2)^(1/2)*
EllipticE(cos(x),I)+2*sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)
^2)^(1/2)
```

3.58.5 Fricas [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos(x)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((cos(x)^2 + 1)^(3/2), x)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((1+cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx=\int \left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 + 1)^(3/2), x)
```

3.58.8 Giac [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx=\int \left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((cos(x)^2 + 1)^(3/2), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx=\int \left(\cos\left(x\right)^2+1\right)^{3/2}\,dx$$

input
```
int((cos(x)^2 + 1)^(3/2),x)
```

output
```
int((cos(x)^2 + 1)^(3/2), x)
```

3.59 $\int (-1-\cos^2(x))^{3/2}\,dx$

3.59.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int (-1-\cos^2(x))^{3/2}\,dx = -\frac{2\sqrt{-1-\cos^2(x)}E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}} - \frac{2\sqrt{1+\cos^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)}{3\sqrt{-1-\cos^2(x)}} - \frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\sin(x)$$

output
```
-1/3*cos(x)*sin(x)*(-1-cos(x)^2)^(1/2)+2*(sin(x)^2)^(1/2)/sin(x)*EllipticE
(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^2)^(1/2)+2/3*(sin(x)^2)^(1/2)/sin
(x)*EllipticF(cos(x),I)*(1+cos(x)^2)^(1/2)/(-1-cos(x)^2)^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int (-1-\cos^2(x))^{3/2}\,dx = \frac{48\sqrt{3+\cos(2x)}E\left(x\middle|\frac{1}{2}\right)-8\sqrt{3+\cos(2x)}\,\text{EllipticF}\left(x,\frac{1}{2}\right)+6\sin(2x)+\sin(4x)}{12\sqrt{2}\sqrt{-3-\cos(2x)}}$$

input
```
Integrate[(-1 - Cos[x]^2)^(3/2),x]
```

output
```
(48*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] - 8*Sqrt[3 + Cos[2*x]]*EllipticF[
x, 1/2] + 6*Sin[2*x] + Sin[4*x])/(12*Sqrt[2]*Sqrt[-3 - Cos[2*x]])
```

3.59.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\cos^2(x)-1\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sin\left(x+\frac{\pi}{2}\right)^2-1\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2\left(3\cos^2(x)+2\right)}{\sqrt{-\cos^2(x)-1}}dx-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3\cos^2(x)+2}{\sqrt{-\cos^2(x)-1}}dx-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{3\sin\left(x+\frac{\pi}{2}\right)^2+2}{\sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \text{3651}$$

$$\frac{2}{3}\left(-\int \frac{1}{\sqrt{-\cos^2(x)-1}}dx-3\int \sqrt{-\cos^2(x)-1}dx\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(-\int \frac{1}{\sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx-3\int \sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}dx\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \text{3657}$$

$$\frac{2}{3}\left(-\int \frac{1}{\sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx-\frac{3\sqrt{-\cos^2(x)-1}\int \sqrt{\cos^2(x)+1}dx}{\sqrt{\cos^2(x)+1}}\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{3}\left(-\int \frac{1}{\sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx-\frac{3\sqrt{-\cos^2(x)-1}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cos^2(x)+1}}\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \; 3656$$

$$\frac{2}{3}\left(-\int \frac{1}{\sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx-\frac{3\sqrt{-\cos^2(x)-1}E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \; 3662$$

$$\frac{2}{3}\left(-\frac{\sqrt{\cos^2(x)+1}\int \frac{1}{\sqrt{\cos^2(x)+1}}dx}{\sqrt{-\cos^2(x)-1}}-\frac{3\sqrt{-\cos^2(x)-1}E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{3}\left(-\frac{\sqrt{\cos^2(x)+1}\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{-\cos^2(x)-1}}-\frac{3\sqrt{-\cos^2(x)-1}E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

$$\Big\downarrow \; 3661$$

$$\frac{2}{3}\left(-\frac{\sqrt{\cos^2(x)+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)}{\sqrt{-\cos^2(x)-1}}-\frac{3\sqrt{-\cos^2(x)-1}E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}\right)-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}$$

input
```
Int[(-1 - Cos[x]^2)^(3/2),x]
```

output
```
(2*((-3*Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2] -
(Sqrt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/Sqrt[-1 - Cos[x]^2]))/3 - (Co
s[x]*Sqrt[-1 - Cos[x]^2]*Sin[x])/3
```

3.59. $\int \left(-1-\cos^2(x)\right)^{3/2}\,dx$

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.59.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result
default	$\frac{\sqrt{-(1+\cos^2(x))(\sin^2(x))}\left(-(\sin^4(x))\cos(x)+10i\sqrt{2-(\sin^2(x))}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}F(i\cos(x),i)-6i\sqrt{2-(\sin^2(x))}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}E(i\cos}{3\sqrt{-1+\cos^4(x)}\sin(x)\sqrt{-1-(\cos^2(x))}}$

input
```
int((-1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(-sin(x)^4*cos(x)+10*I*(2-sin(x)^2)^(1/
2)*(sin(x)^2)^(1/2)*EllipticF(I*cos(x),I)-6*I*(2-sin(x)^2)^(1/2)*(sin(x)^2
)^(1/2)*EllipticE(I*cos(x),I)+2*sin(x)^2*cos(x))/(-1+cos(x)^4)^(1/2)/sin(x
)/(-1-cos(x)^2)^(1/2)
```

3.59.5 Fricas [F]

$$\int \left(-1-\cos^2(x)\right)^{3/2}\,dx=\int \left(-\cos\left(x\right)^2-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(24*(e^(4*I*x) - e^(3*I*x))*integral(-4/3*sqrt(e^(4*I*x) + 6*e^(2*I*x
) + 1)*(5*e^(2*I*x) + 2*e^(I*x) + 5)/(e^(6*I*x) - 2*e^(5*I*x) + 7*e^(4*I*x
) - 12*e^(3*I*x) + 7*e^(2*I*x) - 2*e^(I*x) + 1), x) - (e^(5*I*x) - e^(4*I*
x) + 24*e^(3*I*x) + 24*e^(2*I*x) - e^(I*x) + 1)*sqrt(e^(4*I*x) + 6*e^(2*I*
x) + 1))/(e^(4*I*x) - e^(3*I*x))
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \left(-1-\cos^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-1-cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.59.7 Maxima [F]

$$\int \left(-1-\cos^2(x)\right)^{3/2} dx = \int \left(-\cos(x)^2-1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-cos(x)^2 - 1)^(3/2), x)
```

3.59.8 Giac [F]

$$\int \left(-1-\cos^2(x)\right)^{3/2} dx = \int \left(-\cos(x)^2-1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-cos(x)^2 - 1)^(3/2), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1-\cos^2(x)\right)^{3/2} dx = \int \left(-\cos(x)^2-1\right)^{3/2} dx$$

input
```
int((- cos(x)^2 - 1)^(3/2),x)
```

output
```
int((- cos(x)^2 - 1)^(3/2), x)
```

3.60 $\int (a+b\cos^2(x))^{3/2}\,dx$

3.60.1 Optimal result

Integrand size = 12, antiderivative size = 121

$$\int (a+b\cos^2(x))^{3/2}\,dx = \frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{1+\frac{b\cos^2(x)}{a}}} - \frac{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}} + \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\sin(x)$$

output
```
1/3*b*cos(x)*sin(x)*(a+b*cos(x)^2)^(1/2)-2/3*(2*a+b)*(sin(x)^2)^(1/2)/sin(
x)*EllipticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2)/(1+b*cos(x)^2/a)^(1
/2)+1/3*a*(a+b)*(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),(-b/a)^(1/2))*(1+
b*cos(x)^2/a)^(1/2)/(a+b*cos(x)^2)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int (a+b\cos^2(x))^{3/2}\,dx = \frac{8(2a^2+3ab+b^2)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right) - 4a(a+b)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,\text{EllipticF}\left(x,\frac{b}{a+b}\right)}{12\sqrt{2a+b+b\cos(2x)}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(3/2),x]
```

output
```
(8*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x,
 b/(a + b)] - 4*a*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x
, b/(a + b)] + Sqrt[2]*b*(2*a + b + b*Cos[2*x])*Sin[2*x])/(12*Sqrt[2*a + b
 + b*Cos[2*x]])
```

3.60.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cos^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\cos^2(x)+a(3a+b)}{\sqrt{b\cos^2(x)+a}}dx+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2+a(3a+b)}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\cos^2(x)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\cos^2(x)+a}}dx\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3657}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3662}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\cos^2(x)}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}\right)+$$
$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}\right)+$$
$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3661}$$

3.60. $\int \left(a+b\cos^2(x)\right)^{3/2}\,dx$

$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}+\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}\right)$$

input
```
Int[(a + b*Cos[x]^2)^(3/2),x]
```

output
```
((2*(2*a + b)*Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (
b*Cos[x]^2)/a] - (a*(a + b)*Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -
(b/a)])/Sqrt[a + b*Cos[x]^2])/3 + (b*Cos[x]*Sqrt[a + b*Cos[x]^2]*Sin[x])/3
```

3.60.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.60.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.59

method	result
default	$-\frac{-\frac{a^{2}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^{2}(x)\right)}{a}}\,F\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{3}-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^{2}(x)\right)}{a}}\,F\left(\cos(x),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^{2}(x)\right)}{a}}\,E}{3}}{\sin(x)\sqrt{a+b(\cos}}$

input
```
int((a+b*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3*a^2*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-b/
a)^(1/2))-1/3*a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x)
,(-b/a)^(1/2))*b+4/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(c
os(x),(-b/a)^(1/2))*a^2+2/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*Elli
pticE(cos(x),(-b/a)^(1/2))*a*b+1/3*cos(x)^5*b^2+1/3*b*cos(x)^3*a-1/3*b^2*c
os(x)^3-1/3*b*cos(x)*a)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

3.60.5 Fricas [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(x)^2 + a)^(3/2), x)
```

3.60.6 Sympy [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(a+b\cos^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cos(x)**2)**(3/2), x)
```

3.60.7 Maxima [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(b\cos\left(x\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(x)^2 + a)^(3/2), x)
```

3.60.8 Giac [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(b\cos\left(x\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(x)^2 + a)^(3/2), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*cos(x)^2)^(3/2),x)
```

output
```
int((a + b*cos(x)^2)^(3/2), x)
```

3.61 $\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$

3.61.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \operatorname{EllipticF}\left(\frac{\pi}{2}+x,-1\right)$$

output
```
-(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),I)
```

3.61.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \frac{\operatorname{EllipticF}\left(x,\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[1 + Cos[x]^2],x]
```

output
```
EllipticF[x, 1/2]/Sqrt[2]
```

3.61.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos^2(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx$$

$$\Big\downarrow \text{3661}$$

$$\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)$$

input
```
Int[1/Sqrt[1 + Cos[x]^2],x]
```

output
```
EllipticF[Pi/2 + x, -1]
```

3.61.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

3.61.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.56

method	result	size
default	$-\frac{\sqrt{(1+\cos^2(x))(\sin^2(x))}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,F(\cos(x),i)}{\sqrt{1-(\cos^4(x))}\,\sin(x)}$	41

input
```
int(1/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(1-cos(x)^4)^(1/2)*Ellipti
cF(cos(x),I)/sin(x)
```

3.61.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 9.67

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \sqrt{2\sqrt{2}-3}\left(2i\sqrt{2}+3i\right)F\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos(x)+i\sin(x))\right)\,|\,12\sqrt{2}+17\right) + \sqrt{2\sqrt{2}-3}\left(-2i\sqrt{2}-3i\right)F\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos(x)-i\sin(x))\right)\,|\,12\sqrt{2}+17\right)$$

input
```
integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*sqrt(2) - 3)*(2*I*sqrt(2) + 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) -
 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + sqrt(2*sqrt(2) - 3)*(-2*I*sqr
t(2) - 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12
*sqrt(2) + 17)
```

3.61.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos^2{(x)}+1}}\,dx$$

input
```
integrate(1/(1+cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(cos(x)**2 + 1), x)
```

3.61.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos{(x)}^2+1}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cos(x)^2 + 1), x)
```

3.61.8 Giac [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos{(x)}^2+1}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cos(x)^2 + 1), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos(x)^2+1}}\,dx$$

input
```
int(1/(cos(x)^2 + 1)^(1/2),x)
```

output
```
int(1/(cos(x)^2 + 1)^(1/2), x)
```

3.62 $\int \frac{1}{\sqrt{-1-\cos^2(x)}} \, dx$

3.62.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}} \, dx = \frac{\sqrt{1+\cos^2(x)} \operatorname{EllipticF}\left(\frac{\pi}{2}+x,-1\right)}{\sqrt{-1-\cos^2(x)}}$$

output
```
-(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),I)*(1+cos(x)^2)^(1/2)/(-1-cos(x)
^2)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}} \, dx = \frac{\sqrt{3+\cos(2x)} \operatorname{EllipticF}\left(x,\frac{1}{2}\right)}{\sqrt{2}\sqrt{-3-\cos(2x)}}$$

input
```
Integrate[1/Sqrt[-1 - Cos[x]^2],x]
```

output
```
(Sqrt[3 + Cos[2*x]]*EllipticF[x, 1/2])/(Sqrt[2]*Sqrt[-3 - Cos[2*x]])
```

3.62.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cos^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\cos^2(x)+1}\int \frac{1}{\sqrt{\cos^2(x)+1}}dx}{\sqrt{-\cos^2(x)-1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos^2(x)+1}\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{-\cos^2(x)-1}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\cos^2(x)+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)}{\sqrt{-\cos^2(x)-1}}$$

input
```
Int[1/Sqrt[-1 - Cos[x]^2],x]
```

output
```
(Sqrt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/Sqrt[-1 - Cos[x]^2]
```

3.62.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

method	result	size
default	$\frac{i\sqrt{-\left(1+\cos^{2}\left(x\right)\right)\left(\sin^{2}\left(x\right)\right)}\,\sqrt{1+\cos^{2}\left(x\right)}\,\sqrt{\frac{1}{2}-\frac{\cos\left(2x\right)}{2}}\,F\left(i\cos\left(x\right),i\right)}{\sqrt{-1+\cos^{4}\left(x\right)}\,\sin\left(x\right)\sqrt{-1-\left(\cos^{2}\left(x\right)\right)}}$	62

input
```
int(1/(-1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(-1+c
os(x)^4)^(1/2)*EllipticF(I*cos(x),I)/sin(x)/(-1-cos(x)^2)^(1/2)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx = 2\left(2\sqrt{2}+3\right)\sqrt{2\sqrt{2}-3}F(\arcsin\left(\sqrt{2\sqrt{2}-3}e^{(i\,x)}\right)\mid 12\sqrt{2}+17)$$

input
```
integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*(2*sqrt(2) + 3)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3
)*e^(I*x)), 12*sqrt(2) + 17)
```

3.62.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cos^2(x)-1}}\,dx$$

input
```
integrate(1/(-1-cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-cos(x)**2 - 1), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cos(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-cos(x)^2 - 1), x)
```

3.62.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cos(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-cos(x)^2 - 1), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cos(x)^2-1}}\,dx$$

input
```
int(1/(- cos(x)^2 - 1)^(1/2),x)
```

output
```
int(1/(- cos(x)^2 - 1)^(1/2), x)
```

3.63 $\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx$

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx=\frac{\sqrt{1+\frac{b\cos^2(x)}{a}}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

output
```
-(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),(-b/a)^(1/2))*(1+b*cos(x)^2/a)^(
1/2)/(a+b*cos(x)^2)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx=\frac{\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,\text{EllipticF}\left(x,\frac{b}{a+b}\right)}{\sqrt{2a+b+b\cos(2x)}}$$

input
```
Integrate[1/Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x, b/(a + b)])/Sqrt[2*a +
b + b*Cos[2*x]]
```

3.63.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\cos^2(x)}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

input
```
Int[1/Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -(b/a)])/Sqrt[a + b*Cos[x]^2
]
```

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\left(\cos^2(x)\right)}{a}}\,F\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b(\cos^2(x))}}$	48

input
```
int(1/(a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-b/a)^(1/2))/
sin(x)/(a+b*cos(x)^2)^(1/2)
```

3.63.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 6.57

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx =$$

$$-\frac{\left(-2i\,b\sqrt{\frac{a^2+ab}{b^2}}-2i\,a-i\,b\right)\sqrt{b}\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}-2\,a-b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}-2\,a-b}{b}}(\cos(x)+i\,\sin(x))\right)\,|\,\frac{8\,a^2-\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((-2*I*b*sqrt((a^2 + a*b)/b^2) - 2*I*a - I*b)*sqrt(b)*sqrt((2*b*sqrt((a^2
 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^
2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b
^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*I*b*sqrt((a^2 + a*b)/b^2) + 2*I*a + I
*b)*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsi
n(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) - I*sin(x))), (8*a
^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/b^2
```

3.63.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cos^2{(x)}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cos(x)**2), x)
```

3.63.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cos{(x)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cos(x)^2 + a), x)
```

3.63.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cos{(x)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cos{(x)}^2+a}}\,dx$$

input
```
int(1/(a + b*cos(x)^2)^(1/2),x)
```

output
```
int(1/(a + b*cos(x)^2)^(1/2), x)
```

3.64 $\int \frac{1}{\left(1+\cos^2(x)\right)^{3/2}}\,dx$

3.64.1 Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{1}{\left(1+\cos^2(x)\right)^{3/2}}\,dx=\frac{1}{2}E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}$$

output
```
-1/2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)-1/2*cos(x)*sin(x)/(1+cos(
x)^2)^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(1+\cos^2(x)\right)^{3/2}}\,dx=\frac{E\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}}-\frac{\sin(2x)}{2\sqrt{2}\sqrt{3+\cos(2x)}}$$

input
```
Integrate[(1 + Cos[x]^2)^(-3/2),x]
```

output
```
EllipticE[x, 1/2]/Sqrt[2] - Sin[2*x]/(2*Sqrt[2]*Sqrt[3 + Cos[2*x]])
```

3.64.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3663, 25, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos^2(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{2}\int -\sqrt{\cos^2(x)+1}dx-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \sqrt{\cos^2(x)+1}dx-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{2}E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

input
```
Int[(1 + Cos[x]^2)^(-3/2),x]
```

output
```
EllipticE[Pi/2 + x, -1]/2 - (Cos[x]*Sin[x])/(2*Sqrt[1 + Cos[x]^2])
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.64.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(32) = 64$.

Time = 0.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

method	result	size
default	$-\frac{\sqrt{-\left(\sin^4(x)\right)+2\left(\sin^2(x)\right)}\,\left(\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{2-\left(\sin^2(x)\right)}\,E(\cos(x),i)+\left(\sin^2(x)\right)\cos(x)\right)}{2\sqrt{1-\left(\cos^4(x)\right)}\,\sin(x)\sqrt{1+\cos^2(x)}}$	70

input
```
int(1/(1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-sin(x)^4+2*sin(x)^2)^(1/2)*((sin(x)^2)^(1/2)*(2-sin(x)^2)^(1/2)*Ell
ipticE(cos(x),I)+sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)^2)^(
1/2)
```

3.64.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 7.72

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \frac{\left((2i\sqrt{2}-3i)\cos(x)^2+2i\sqrt{2}-3i\right)\sqrt{2\sqrt{2}-3}E\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos(x)+i\,\mathrm{s}\right.\right.}{}$$

input
```
integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(((2*I*sqrt(2) - 3*I)*cos(x)^2 + 2*I*sqrt(2) - 3*I)*sqrt(2*sqrt(2) - 3
)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) +
 17) + ((-2*I*sqrt(2) + 3*I)*cos(x)^2 - 2*I*sqrt(2) + 3*I)*sqrt(2*sqrt(2)
- 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(2
) + 17) - 4*((-I*sqrt(2) - 3*I)*cos(x)^2 - I*sqrt(2) - 3*I)*sqrt(2*sqrt(2)
 - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(
2) + 17) - 4*((I*sqrt(2) + 3*I)*cos(x)^2 + I*sqrt(2) + 3*I)*sqrt(2*sqrt(2)
 - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(
2) + 17) - 2*sqrt(cos(x)^2 + 1)*cos(x)*sin(x))/(cos(x)^2 + 1)
```

3.64.6 Sympy [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{(\cos^2(x)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)**2)**(3/2),x)
```

output
```
Integral((cos(x)**2 + 1)**(-3/2), x)
```

3.64.7 Maxima [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 + 1)^(-3/2), x)
```

3.64.8 Giac [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((cos(x)^2 + 1)^(-3/2), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{3/2}}\,dx$$

input
```
int(1/(cos(x)^2 + 1)^(3/2),x)
```

output
```
int(1/(cos(x)^2 + 1)^(3/2), x)
```

3.65 $\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx$

3.65.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx = \frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{2\sqrt{1+\cos^2(x)}}+\frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}$$

output
```
1/2*cos(x)*sin(x)/(-1-cos(x)^2)^(1/2)-1/2*(sin(x)^2)^(1/2)/sin(x)*Elliptic
E(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^2)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx = \frac{-2\sqrt{3+\cos(2x)}\,E\left(x\middle|\frac{1}{2}\right)+\sin(2x)}{2\sqrt{2}\sqrt{-3-\cos(2x)}}$$

input
```
Integrate[(-1 - Cos[x]^2)^(-3/2),x]
```

output
```
(-2*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] + Sin[2*x])/(2*Sqrt[2]*Sqrt[-3 -
Cos[2*x]])
```

3.65.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3663, 2011, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-\cos^2(x)-1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-\sin\left(x+\frac{\pi}{2}\right)^2-1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}-\frac{1}{2}\int \frac{\cos^2(x)+1}{\sqrt{-\cos^2(x)-1}}dx$$

$$\Big\downarrow \text{2011}$$

$$\frac{1}{2}\int \sqrt{-\cos^2(x)-1}dx+\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{-\sin\left(x+\frac{\pi}{2}\right)^2-1}dx+\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{-\cos^2(x)-1}\int \sqrt{\cos^2(x)+1}dx}{2\sqrt{\cos^2(x)+1}}+\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cos^2(x)-1}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{2\sqrt{\cos^2(x)+1}}+\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}+\frac{\sqrt{-\cos^2(x)-1}E\left(x+\frac{\pi}{2}\middle|-1\right)}{2\sqrt{\cos^2(x)+1}}$$

input
```
Int[(-1 - Cos[x]^2)^(-3/2),x]
```

output
```
(Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/(2*Sqrt[1 + Cos[x]^2]) + (Co
s[x]*Sin[x])/(2*Sqrt[-1 - Cos[x]^2])
```

3.65.3.1 Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.65.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{\sin^4(x)-2(\sin^2(x))}\left(2i\sqrt{2-(\sin^2(x))}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,F(i\cos(x),i)-i\sqrt{2-(\sin^2(x))}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,E(i\cos(x),i)-(\sin^2(x))\cos(x)\right.}{2\sqrt{-1+\cos^4(x)}\,\sin(x)\sqrt{-1-(\cos^2(x))}}$

input
```
int(1/(-1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(sin(x)^4-2*sin(x)^2)^(1/2)*(2*I*(sin(x)^2)^(1/2)*EllipticF(I*cos(x),
I)*(2-sin(x)^2)^(1/2)-I*(sin(x)^2)^(1/2)*EllipticE(I*cos(x),I)*(2-sin(x)^2
)^(1/2)-sin(x)^2*cos(x))/(-1+cos(x)^4)^(1/2)/sin(x)/(-1-cos(x)^2)^(1/2)
```

3.65.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(51) = 102.

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.00

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx =$$
$$-\frac{\left((2\sqrt{2}-3)e^{(4ix)}+6\left(2\sqrt{2}-3\right)e^{(2ix)}+2\sqrt{2}-3\right)\sqrt{2\sqrt{2}-3}E(\arcsin\left(\sqrt{2\sqrt{2}-3}e^{(ix)}\right)\mid 12\sqrt{2}+17)}{}$$

input
```
integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((2*sqrt(2) - 3)*e^(4*I*x) + 6*(2*sqrt(2) - 3)*e^(2*I*x) + 2*sqrt(2)
 - 3)*sqrt(2*sqrt(2) - 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*e^(I*x)),
12*sqrt(2) + 17) + 4*((sqrt(2) + 3)*e^(4*I*x) + 6*(sqrt(2) + 3)*e^(2*I*x)
+ sqrt(2) + 3)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*e
^(I*x)), 12*sqrt(2) + 17) + sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(e^(3*I*x) +
 3*e^(I*x)))/(e^(4*I*x) + 6*e^(2*I*x) + 1)
```

3.65.6 Sympy [F]

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-\cos^2{(x)}-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-1-cos(x)**2)**(3/2),x)
```

output
```
Integral((-cos(x)**2 - 1)**(-3/2), x)
```

3.65.7 Maxima [F]

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-\cos\left(x\right)^2-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-cos(x)^2 - 1)^(-3/2), x)
```

3.65.8 Giac [F]

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-\cos\left(x\right)^2-1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-cos(x)^2 - 1)^(-3/2), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(-\cos\left(x\right)^2-1\right)^{3/2}}\,dx$$

input
```
int(1/(- cos(x)^2 - 1)^(3/2),x)
```

output
```
int(1/(- cos(x)^2 - 1)^(3/2), x)
```

3.66 $\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx$

3.66.1 Optimal result

Integrand size = 12, antiderivative size = 78

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}}-\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

output
```
-b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^(1/2)-(sin(x)^2)^(1/2)/sin(x)*Elli
pticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2)/a/(a+b)/(1+b*cos(x)^2/a)^(
1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\frac{2(a+b)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right)-\sqrt{2}b\sin(2x)}{2a(a+b)\sqrt{2a+b+b\cos(2x)}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-3/2),x]
```

output
```
(2*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x, b/(a + b)] -
Sqrt[2]*b*Sin[2*x])/(2*a*(a + b)*Sqrt[2*a + b + b*Cos[2*x]])
```

3.66.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\sqrt{b\cos^2(x)+a}dx}{a(a+b)}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\cos^2(x)+a}dx}{a(a+b)}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{a(a+b)}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

input
```
Int[(a + b*Cos[x]^2)^(-3/2),x]
```

output
```
(Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/(a*(a + b)*Sqrt[1 + (b*
Cos[x]^2)/a]) - (b*Cos[x]*Sin[x])/(a*(a + b)*Sqrt[a + b*Cos[x]^2])
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.66.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\frac{b\left(\sin^{2}(x)\right)}{a}+\frac{a+b}{a}}\,aE\left(\cos(x),\sqrt{-\frac{b}{a}}\right)+b\left(\sin^{2}(x)\right)\cos(x)}{a(a+b)\sin(x)\sqrt{a+b\left(\cos^{2}(x)\right)}}$	73

input
```
int(1/(a+b*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*a*EllipticE(cos(x),(-b/a)
^(1/2))+b*sin(x)^2*cos(x))/a/(a+b)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

3.66.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 775, normalized size of antiderivative = 9.94

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{b\cos(x)^2+a}b^3\cos(x)\sin(x)+\left(2i\,a^2b+i\,ab^2+(2i\,ab^2+i\,b^3)\cos(x)^2-2\left(i\,b^3\cos(x)^2+i\,ab^2\right)\sqrt{\frac{a^2+}{b}}\right.}{}$$

input
```
integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(b*cos(x)^2 + a)*b^3*cos(x)*sin(x) + (2*I*a^2*b + I*a*b^2 + (2
*I*a*b^2 + I*b^3)*cos(x)^2 - 2*(I*b^3*cos(x)^2 + I*a*b^2)*sqrt((a^2 + a*b)
/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_e(ar
csin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (
8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*
a^2*b - I*a*b^2 + (-2*I*a*b^2 - I*b^3)*cos(x)^2 - 2*(-I*b^3*cos(x)^2 - I*a
*b^2)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a
 - b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(
cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*
b)/b^2))/b^2) + 2*(-2*I*a^3 - 3*I*a^2*b - I*a*b^2 + (-2*I*a^2*b - 3*I*a*b^
2 - I*b^3)*cos(x)^2 + 2*(-I*a*b^2*cos(x)^2 - I*a^2*b)*sqrt((a^2 + a*b)/b^2
))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin
(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^
2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*I*a^3
 + 3*I*a^2*b + I*a*b^2 + (2*I*a^2*b + 3*I*a*b^2 + I*b^3)*cos(x)^2 + 2*(I*a
*b^2*cos(x)^2 + I*a^2*b)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) - 2*a - b)/b)*(cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(a^3*b^2 + a^2*b^3 + (a^2*b^3 + a*b^4)*c
os(x)^2)
```

3.66.6 Sympy [F]

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\cos^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cos(x)**2)**(-3/2), x)
```

3.66.7 Maxima [F]

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cos{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(x)^2 + a)^(-3/2), x)
```

3.66.8 Giac [F]

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cos{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(b\cos\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*cos(x)^2)^(3/2),x)
```

output
```
int(1/(a + b*cos(x)^2)^(3/2), x)
```

3.67 $\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx$

3.67.1 Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

output
```
arcsin(1/2*sin(x)*2^(1/2))
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

input
```
Integrate[Cos[x]/Sqrt[1 + Cos[x]^2],x]
```

output
```
ArcSin[Sin[x]/Sqrt[2]]
```

3.67.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3665, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\cos^2(x)+1}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx$$
$$\Big\downarrow \text{3665}$$
$$\int \frac{1}{\sqrt{2-\sin^2(x)}}d\sin(x)$$
$$\Big\downarrow \text{223}$$
$$\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

input
```
Int[Cos[x]/Sqrt[1 + Cos[x]^2],x]
```

output
```
ArcSin[Sin[x]/Sqrt[2]]
```

3.67.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.67

method	result	size
default	$-\frac{\sqrt{(1+\cos^2(x))(\sin^2(x))}\,\arcsin(\cos^2(x))}{2\sin(x)\sqrt{1+\cos^2(x)}}$	33

input
```
int(cos(x)/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((1+cos(x)^2)*sin(x)^2)^(1/2)*arcsin(cos(x)^2)/sin(x)/(1+cos(x)^2)^(1
/2)
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 5.44

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\cos(x)^2+1}\cos(x)^2\sin(x) - \cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) + \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan((sqrt(cos(x)^2 + 1)*cos(x)^2*sin(x) - cos(x)*sin(x))/(cos(x)^4
+ cos(x)^2 - 1)) + 1/2*arctan(sin(x)/cos(x))
```

3.67.6 Sympy [F]

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

input
```
integrate(cos(x)/(1+cos(x)**2)**(1/2),x)
```

output
```
Integral(cos(x)/sqrt(cos(x)**2 + 1), x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \arcsin\left(\frac{1}{2}\sqrt{2}\sin(x)\right)$$

input
```
integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(1/2*sqrt(2)*sin(x))
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{1}{2}\sqrt{-\sin(x)^2+2}\sin(x) + \arcsin\left(\frac{1}{2}\sqrt{2}\sin(x)\right)$$

input
```
integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-sin(x)^2 + 2)*sin(x) + arcsin(1/2*sqrt(2)*sin(x))
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\cos(x)^2+1}}\,dx$$

input
```
int(cos(x)/(cos(x)^2 + 1)^(1/2),x)
```

output
```
int(cos(x)/(cos(x)^2 + 1)^(1/2), x)
```

3.68 $\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx=\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(5+3x)\right)$$

output
```
1/3*arcsin(1/2*sin(5+3*x))
```

3.68.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx=\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(5+3x)\right)$$

input
```
Integrate[Cos[5 + 3*x]/Sqrt[3 + Cos[5 + 3*x]^2],x]
```

output
```
ArcSin[Sin[5 + 3*x]/2]/3
```

3.68.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3665, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(3x+5)}{\sqrt{\cos^2(3x+5)+3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(3x+\frac{\pi}{2}+5\right)}{\sqrt{\sin\left(3x+\frac{\pi}{2}+5\right)^2+3}}dx$$

$$\Big\downarrow \ 3665$$

$$\frac{1}{3}\int \frac{1}{\sqrt{4-\sin^2(3x+5)}}d\sin(3x+5)$$

$$\Big\downarrow \ 223$$

$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x+5)\right)$$

input
```
Int[Cos[5 + 3*x]/Sqrt[3 + Cos[5 + 3*x]^2],x]
```

output
```
ArcSin[Sin[5 + 3*x]/2]/3
```

3.68.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.65 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

method	result	size
default	$\frac{\sqrt{(3+\cos^2(5+3x))(\sin^2(5+3x))}\,\arcsin\left(-1+\frac{(\sin^2(5+3x))}{2}\right)}{6\sin(5+3x)\sqrt{3+\cos^2(5+3x)}}$	57

input
```
int(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((3+cos(5+3*x)^2)*sin(5+3*x)^2)^(1/2)*arcsin(-1+1/2*sin(5+3*x)^2)/sin(
5+3*x)/(3+cos(5+3*x)^2)^(1/2)
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.93

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx$$
$$= \frac{1}{6}\arctan\left(\frac{\sqrt{\cos(3x+5)^2+3}\left(\cos(3x+5)^2+1\right)\sin(3x+5)-4\cos(3x+5)\sin(3x+5)}{\cos(3x+5)^4+6\cos(3x+5)^2-3}\right)$$
$$+\frac{1}{6}\arctan\left(\frac{\sin(3x+5)}{\cos(3x+5)}\right)$$

input
```
integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*arctan((sqrt(cos(3*x + 5)^2 + 3)*(cos(3*x + 5)^2 + 1)*sin(3*x + 5) - 4
*cos(3*x + 5)*sin(3*x + 5))/(cos(3*x + 5)^4 + 6*cos(3*x + 5)^2 - 3)) + 1/6
*arctan(sin(3*x + 5)/cos(3*x + 5))
```

3.68.6 Sympy [F]

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx = \int \frac{\cos(3x+5)}{\sqrt{\cos^2(3x+5)+3}}\,dx$$

input
```
integrate(cos(5+3*x)/(3+cos(5+3*x)**2)**(1/2),x)
```

output
```
Integral(cos(3*x + 5)/sqrt(cos(3*x + 5)**2 + 3), x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x+5)\right)$$

input
```
integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsin(1/2*sin(3*x + 5))
```

3.68.8 Giac [F]

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx = \int \frac{\cos(3x+5)}{\sqrt{\cos(3x+5)^2+3}}\,dx$$

input
```
integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(3*x + 5)/sqrt(cos(3*x + 5)^2 + 3), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx = \int \frac{\cos\left(3\,x+5\right)}{\sqrt{{\cos\left(3\,x+5\right)}^2+3}}\,dx$$

input
```
int(cos(3*x + 5)/(cos(3*x + 5)^2 + 3)^(1/2),x)
```

output
```
int(cos(3*x + 5)/(cos(3*x + 5)^2 + 3)^(1/2), x)
```

3.69 $\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx$

3.69.1 Optimal result

Integrand size = 15, antiderivative size = 9

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \operatorname{arcsinh}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

output
```
arcsinh(1/3*sin(x)*3^(1/2))
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \operatorname{arcsinh}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

input
```
Integrate[Cos[x]/Sqrt[4 - Cos[x]^2],x]
```

output
```
ArcSinh[Sin[x]/Sqrt[3]]
```

3.69.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3665, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{\sqrt{4-\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\int \frac{1}{\sqrt{\sin^2(x)+3}}d\sin(x)$$

$$\Big\downarrow \text{ 222}$$

$$\text{arcsinh}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

input
```
Int[Cos[x]/Sqrt[4 - Cos[x]^2],x]
```

output
```
ArcSinh[Sin[x]/Sqrt[3]]
```

3.69.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(8) = 16$.

Time = 0.67 (sec) , antiderivative size = 53, normalized size of antiderivative = 5.89

method	result	size
default	$-\frac{\sqrt{-\left(-4+\cos^2(x)\right)\left(\sin^2(x)\right)}\,\ln\left(-\left(\sin^2(x)\right)+\sqrt{\sin^4(x)+3\left(\sin^2(x)\right)}-\frac{3}{2}\right)}{2\sin(x)\sqrt{4-\left(\cos^2(x)\right)}}$	53

input
```
int(cos(x)/(4-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-4+cos(x)^2)*sin(x)^2)^(1/2)*ln(-sin(x)^2+(sin(x)^4+3*sin(x)^2)^(1
/2)-3/2)/sin(x)/(4-cos(x)^2)^(1/2)
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(8) = 16$.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 4.33

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \frac{1}{4}\log\left(8\cos(x)^4 - 4\left(2\cos(x)^2 - 5\right)\sqrt{-\cos(x)^2+4}\sin(x) - 40\cos(x)^2 + 41\right)$$

input
```
integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 5)*sqrt(-cos(x)^2 + 4)*sin(x) - 40*co
s(x)^2 + 41)
```

3.69.6 Sympy [F]

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{-(\cos(x)-2)(\cos(x)+2)}}\,dx$$

input
```
integrate(cos(x)/(4-cos(x)**2)**(1/2),x)
```

output
```
Integral(cos(x)/sqrt(-(cos(x) - 2)*(cos(x) + 2)), x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}\sin(x)\right)$$

input
```
integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(3)*sin(x))
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(8) = 16.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.22

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \frac{1}{2}\sqrt{\sin(x)^2+3}\sin(x) - \frac{3}{2}\log\left(\sqrt{\sin(x)^2+3}-\sin(x)\right)$$

input
```
integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(sin(x)^2 + 3)*sin(x) - 3/2*log(sqrt(sin(x)^2 + 3) - sin(x))
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{4-\cos(x)^2}}\,dx$$

input
```
int(cos(x)/(4 - cos(x)^2)^(1/2),x)
```

output
```
int(cos(x)/(4 - cos(x)^2)^(1/2), x)
```

3.70 $\int \frac{1}{a+b\cos^4(x)}\,dx$

3.70.1 Optimal result

Integrand size = 10, antiderivative size = 487

$$\int \frac{1}{a+b\cos^4(x)}\,dx = \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\arctan\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}-\sqrt{2}(a+b)^{3/4}\cot(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}$$
$$-\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\arctan\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}+\sqrt{2}(a+b)^{3/4}\cot(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}$$
$$-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\cot(x)+(a+b)^{3/4}\cot^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$
$$+\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\cot(x)+(a+b)^{3/4}\cot^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$

output
```
-1/8*ln((a+b)^(3/4)*cot(x)^2+(a+b)^(1/4)*a^(1/2)-a^(1/4)*cot(x)*2^(1/2)*(a
+b-a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2
^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+1/8*ln((a+b)^(3/4)*cot(x)^2+(a+b)^(
1/4)*a^(1/2)+a^(1/4)*cot(x)*2^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1
/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/
2)+1/4*arctan((-(a+b)^(3/4)*cot(x)*2^(1/2)+a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2
))^(1/2))/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a
^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)-1/4*arctan(((a+
b)^(3/4)*cot(x)*2^(1/2)+a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2))/a^(1/4)/(
a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*
2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.25

$$\int \frac{1}{a+b\cos^4(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}$$

input
```
Integrate[(a + b*Cos[x]^4)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + I*Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + I
*Sqrt[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + I*Sqrt[a]*Sqrt[b]]
]/(2*Sqrt[a]*Sqrt[-a + I*Sqrt[a]*Sqrt[b]])
```

3.70.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3688, 1483, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$-\int \frac{\cot^2(x)+1}{(a+b)\cot^4(x)+2a\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\sqrt[4]{a+b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\cot(x)}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}-\frac{\sqrt[4]{a+b}\int \frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\cot(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int -\frac{\sqrt{2}\left(\sqrt[4]{\cdots}\right.}{\cot^2(x)-\cdots}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\cot(\cdots\right.}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{25}$$

3.70. $\int \frac{1}{a+b\cos^4(x)}\,dx$

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{1}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}\cdots$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{1}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\sqrt{2}\left(\sqrt{2}\cot(\cdots\right.}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{1}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+\cdots}}}{(a+b)^{3/\cdots}}}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(a\cdots}}}{\sqrt{2}\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}\cdots$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int\frac{1}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int\frac{\sqrt{2}\cot(x)+\frac{\sqrt[4]{a}\cdots}{\cdots}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{\cdots}}}{(a+b\cdots}}}{\sqrt{2}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \; 1083$$

3.70. $\int \frac{1}{a+b\cos^4(x)}\,dx$

$$-\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\cot(x)}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\cot(x)-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.\cdots}}{\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\cot(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}}{(a+b)^{3/4}}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\cot(x)+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.\cdots}}{\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 217

$$-\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\cot(x)}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\cot(x)-\cdots\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\right.\cdots}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\cot(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}}{(a+b)^{3/4}}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{(a\cdots}\right.\cdots}{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}\right.\cdots}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 1103

3.70. $\int \frac{1}{a+b\cos^4(x)}\,dx$

$$-\frac{\sqrt[4]{a+b}\left(\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\cot(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}c\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}+2\cot(x)\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}co\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

input
```
Int[(a + b*Cos[x]^4)^(-1),x]
```

output
```
-1/2*((a + b)^(1/4)*(((Sqrt[a] + Sqrt[a + b])*Sqrt[a + b - Sqrt[a]*Sqrt[a
+ b]]*ArcTan[((a + b)^(3/4)*(-((Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[
a + b]])/(a + b)^(3/4)) + 2*Cot[x]))/(Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]
*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((1 - S
qrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b
 - Sqrt[a]*Sqrt[a + b]]*Cot[x] + (a + b)^(3/4)*Cot[x]^2])/2))/(Sqrt[2]*a^(
3/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((a + b)^(1/4)*(((Sqrt[a] + Sqrt
[a + b])*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*ArcTan[((a + b)^(3/4)*((Sqrt[2]
*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])/(a + b)^(3/4) + 2*Cot[x]))/(Sq
rt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b
 + Sqrt[a]*Sqrt[a + b]]) + ((1 - Sqrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^
(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*Cot[x] + (a + b)
^(3/4)*Cot[x]^2])/2))/(2*Sqrt[2]*a^(3/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
)
```

3.70. $\int \frac{1}{a+b\cos^4(x)}\,dx$

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.70.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.21

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^4+256a^3b)\_Z^4+32a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{128ia^4}{b}+128ia^3\right)\_R^3+\left(-\frac{32a^3}{b}-32a^2\right)\_R$
default	Expression too large to display

input
```
int(1/(a+b*cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*x)+(128*I/b*a^4+128*I*a^3)*_R^3+(-32/b*a^3-32*a^2)*_R^2+
(8*I/b*a^2-8*I*a)*_R-2/b*a+1),_R=RootOf(1+(256*a^4+256*a^3*b)*_Z^4+32*a^2*
_Z^2))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(344) = 688.

Time = 0.35 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.66

$$\int \frac{1}{a+b\cos^4(x)}\,dx = -\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\Bigg(b\cos(x)^2$$

$$+2\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+}{a^2+ab}}$$

$$-\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\Bigg)$$

$$+\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\Bigg(b\cos(x)^2$$

$$-2\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+}{a^2+ab}}$$

$$-\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\Bigg)$$

$$+\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\Bigg(-b\cos(x)^2$$

$$+2\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}$$

$$-\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\Bigg)$$

$$-\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\Bigg(-b\cos(x)^2$$

$$-2\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}$$

$$-\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\Bigg)$$

input
```
integrate(1/(a+b*cos(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b
))*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*
a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b
 + a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*
sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5
+ 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin
(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt
(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3
 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) +
 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)
)*log(-b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*
a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b
+ a^3*b^2)) - 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*s
qrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 +
2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(
x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(
((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - (a^3 +
 a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**4),x)
```

output
```
Timed out
```

3.70.7 Maxima [F]

$$\int \frac{1}{a+b\cos^4(x)}\,dx = \int \frac{1}{b\cos{(x)}^4+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^4 + a), x)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.63

$$\int \frac{1}{a+b\cos^4(x)}\,dx$$
$$= \frac{\left(3\sqrt{a^2+\sqrt{-ab}a}a^2+4\sqrt{a^2+\sqrt{-ab}a}ab-3\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a-4\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\right.\right.}{2\left(3a^5+7a^4b+4a^3b^2\right)}$$
$$+\frac{\left(3\sqrt{a^2-\sqrt{-ab}a}a^2+4\sqrt{a^2-\sqrt{-ab}a}ab-3\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a-4\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}\right.\right.}{2\left(3a^5+7a^4b+4a^3b^2\right)}$$

input
```
integrate(1/(a+b*cos(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b - 3*s
qrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*
b)*b)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)
*a + 16*a^2))/a)))*abs(a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2
- sqrt(-a*b)*a)*a^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b - 3*sqrt(a^2 - sqrt(-
a*b)*a)*sqrt(-a*b)*a - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*(pi*floor(
x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/a)
))*abs(a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.90

$$\int \frac{1}{a+b\cos^4(x)}\,dx$$
$$= -2\,\mathrm{atanh}\left(\frac{8\,a^6\,b\tan(x)\,\sqrt{-\frac{a^2}{16\,(a^4+b\,a^3)}-\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}}{\frac{2\,a^9\,b}{a^4+b\,a^3}-2\,a^4\,b^2-2\,a^5\,b+\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} - \frac{8\,a^2\,b\tan(x)\,\sqrt{-\frac{a^2}{16\,(a^4+b\,a^3)}-\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}}{\frac{2\,a^5\,b}{a^4+b\,a^3}-2\,a\,b+\frac{2\,a^3\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} + \frac{8\,a^4\,b\tan(x)\,\sqrt{-a^3\,b}\,\sqrt{-\frac{a^2}{16\,(a^4+b\,a^3)}-\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}}{\frac{2\,a^9\,b}{a^4+b\,a^3}-2\,a^4\,b^2-2\,a^5\,b+\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}}\right)\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}$$
$$- 2\,\mathrm{atanh}\left(\frac{8\,a^2\,b\tan(x)\,\sqrt{\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}-\frac{a^2}{16\,(a^4+b\,a^3)}}}{2\,a\,b-\frac{2\,a^5\,b}{a^4+b\,a^3}+\frac{2\,a^3\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} - \frac{8\,a^6\,b\tan(x)\,\sqrt{\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}-\frac{a^2}{16\,(a^4+b\,a^3)}}}{2\,a^5\,b+2\,a^4\,b^2-\frac{2\,a^9\,b}{a^4+b\,a^3}-\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} + \frac{8\,a^4\,b\tan(x)\,\sqrt{-a^3\,b}\,\sqrt{\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}-\frac{a^2}{16\,(a^4+b\,a^3)}}}{2\,a^5\,b+2\,a^4\,b^2-\frac{2\,a^9\,b}{a^4+b\,a^3}-\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}}\right)\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}$$

input
```
int(1/(a + b*cos(x)^4),x)
```

output
```
- 2*atanh((8*a^6*b*tan(x)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(
a^3*b + a^4)))^(1/2))/((2*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*
a^8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b
^2*(-a^3*b)^(1/2))/(a^3*b + a^4)) - (8*a^2*b*tan(x)*(- a^2/(16*(a^3*b + a^
4)) - (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/2))/((2*a^5*b)/(a^3*b + a^4) -
 2*a*b + (2*a^3*b*(-a^3*b)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b
)^(1/2)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/
2))/((2*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*a^8*b^2)/(a^3*b +
a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))
/(a^3*b + a^4)))*(-(a^2 + (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2) - 2*at
anh((8*a^2*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b +
a^4)))^(1/2))/(2*a*b - (2*a^5*b)/(a^3*b + a^4) + (2*a^3*b*(-a^3*b)^(1/2))/
(a^3*b + a^4)) - (8*a^6*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/
(16*(a^3*b + a^4)))^(1/2))/(2*a^5*b + 2*a^4*b^2 - (2*a^9*b)/(a^3*b + a^4)
- (2*a^8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*
a^6*b^2*(-a^3*b)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b)^(1/2)*((
-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b + a^4)))^(1/2))/(2*a^5*b
 + 2*a^4*b^2 - (2*a^9*b)/(a^3*b + a^4) - (2*a^8*b^2)/(a^3*b + a^4) + (2*a^
7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))/(a^3*b + a^
4)))*(-(a^2 - (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2)
```

3.70. $\int \frac{1}{a+b\cos^4(x)}\,dx$

3.71 $\int \frac{1}{a-b\cos^4(x)}\,dx$

3.71.1 Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{1}{a-b\cos^4(x)}\,dx=-\frac{\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

output
```
-1/2*arctan(cot(x)*(a^(1/2)-b^(1/2))^(1/2)/a^(1/4))/a^(3/4)/(a^(1/2)-b^(1/
2))^(1/2)-1/2*arctan(cot(x)*(a^(1/2)+b^(1/2))^(1/2)/a^(1/4))/a^(3/4)/(a^(1
/2)+b^(1/2))^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{1}{a-b\cos^4(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}$$

input
```
Integrate[(a - b*Cos[x]^4)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + Sqr
t[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*S
qrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]])
```

3.71.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3688, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$-\int \frac{\cot^2(x)+1}{(a-b)\cot^4(x)+2a\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{1480}$$

$$-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{(a-b)\cot^2(x)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\cot(x)-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{(a-b)\cot^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\cot(x)$$

$$\Big\downarrow \text{218}$$

$$-\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[(a - b*Cos[x]^4)^(-1),x]
```

output
```
-1/2*((1 + Sqrt[b]/Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Cot[x])/a^(1/4
)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) - ((1 - Sqrt[b]/
Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Cot[x])/a^(1/4)])/(2*a^(1/4)*(Sqr
t[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]])
```

3.71.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.71.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

method	result
default	$a\left(\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2a\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{\operatorname{arctanh}\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2a\sqrt{\left(\sqrt{ab}-a\right)a}}\right)$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(256a^4-256a^3b\right)\_Z^4+32a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{128ia^4}{b}+128ia^3\right)\_R^3+\left(\frac{32a^3}{b}-32a^2\right)\_R\right.$

input
```
int(1/(a-b*cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2/a/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*tan(x)/(((a*b)^(1/2)+a)*a)^(1/
2))-1/2/a/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*tan(x)/(((a*b)^(1/2)-a)*a)^(
1/2)))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 817, normalized size of antiderivative = 8.09

$$\int \frac{1}{a-b\cos^4(x)}\,dx = -\frac{1}{8}\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}+1}{a^2-ab}}\log\Bigg(b\cos(x)^2 +2\left(ab\cos(x)\sin(x)-(a^4-a^3b)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}+1}{a^2-ab}} + \left(a^3-a^2b-2\left(a^3-a^2b\right)\cos(x)^2\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\Bigg)$$

$$+\frac{1}{8}\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}+1}{a^2-ab}}\log\Bigg(b\cos(x)^2 -2\left(ab\cos(x)\sin(x)-(a^4-a^3b)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}+1}{a^2-ab}} + \left(a^3-a^2b-2\left(a^3-a^2b\right)\cos(x)^2\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\Bigg)$$

$$+\frac{1}{8}\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}-1}{a^2-ab}}\log\Bigg(-b\cos(x)^2 +2\left(ab\cos(x)\sin(x)+(a^4-a^3b)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}-1}{a^2-ab}} + \left(a^3-a^2b-2\left(a^3-a^2b\right)\cos(x)^2\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\Bigg)$$

$$-\frac{1}{8}\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}-1}{a^2-ab}}\log\Bigg(-b\cos(x)^2 -2\left(ab\cos(x)\sin(x)+(a^4-a^3b)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}-1}{a^2-ab}} + \left(a^3-a^2b-2\left(a^3-a^2b\right)\cos(x)^2\right)\sqrt{\frac{b}{a^5-2a^4b+a^3b^2}}\Bigg)$$

input
```
integrate(1/(a-b*cos(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)
)*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^
4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b +
a^3*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqr
t(b/(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a
^4*b + a^3*b^2)) + 1)/(a^2 - a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) -
 (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^
2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*
b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqr
t(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(-b*
cos(x)^2 + 2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^
3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))
 - 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5
- 2*a^4*b + a^3*b^2))) - 1/8*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3
*b^2)) - 1)/(a^2 - a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) + (a^4 - a
^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*s
qrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3
 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - b\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cos(x)**4),x)
```

output
```
Timed out
```

3.71.7 Maxima [F]

$$\int \frac{1}{a-b\cos^4(x)}\,dx = \int -\frac{1}{b\cos{(x)}^4-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^4),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^4 - a), x)
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(65) = 130.

Time = 0.38 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.96

$$\int \frac{1}{a-b\cos^4(x)}\,dx$$
$$= \frac{\left(3\sqrt{a^2+\sqrt{ab}a}a^2-4\sqrt{a^2+\sqrt{ab}a}ab-3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a+4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\mathrm{arcta}\right.}{2\left(3a^5-7a^4b+4a^3b^2\right)}$$
$$+\frac{\left(3\sqrt{a^2-\sqrt{ab}a}a^2-4\sqrt{a^2-\sqrt{ab}a}ab+3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\mathrm{arc}\right.}{2\left(3a^5-7a^4b+4a^3b^2\right)}$$

input
```
integrate(1/(a-b*cos(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2 - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b - 3*sqr
t(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b)*
(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a - b)*a + 1
6*a^2))/a)))*abs(a)/(3*a^5 - 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt
(a*b)*a)*a^2 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b + 3*sqrt(a^2 - sqrt(a*b)*a)*s
qrt(a*b)*a - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*(pi*floor(x/pi + 1/2)
+ arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/a)))*abs(a)/(3
*a^5 - 7*a^4*b + 4*a^3*b^2)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 938, normalized size of antiderivative = 9.29

$$\int \frac{1}{a-b\cos^4(x)}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{8\,a^6\,b\tan(x)\,\sqrt{\frac{a^2}{16\,(a^3\,b-a^4)}+\frac{\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}}{2\,a^5\,b-2\,a^4\,b^2-\frac{2\,a^8\,b^2}{a^3\,b-a^4}+\frac{2\,a^9\,b}{a^3\,b-a^4}-\frac{2\,a^6\,b^2\,\sqrt{a^3\,b}}{a^3\,b-a^4}+\frac{2\,a^7\,b\,\sqrt{a^3\,b}}{a^3\,b-a^4}}-\frac{8\,a^2\,b\tan(x)\,\sqrt{\frac{a^2}{16\,(a^3\,b-a^4)}+\frac{\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}}{2\,a\,b+\frac{2\,a^5\,b}{a^3\,b-a^4}+\frac{2\,a^3\,b\,\sqrt{a^3\,b}}{a^3\,b-a^4}}+\frac{8\,a^4\,b\tan(x)\,\sqrt{\frac{a^2}{16\,(a^3\,b-a^4)}+\frac{\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}\,\sqrt{a^3\,b}}{2\,a^5\,b-2\,a^4\,b^2-\frac{2\,a^8\,b^2}{a^3\,b-a^4}+\frac{2\,a^9\,b}{a^3\,b-a^4}-\frac{2\,a^6\,b^2\,\sqrt{a^3\,b}}{a^3\,b-a^4}+\frac{2\,a^7\,b\,\sqrt{a^3\,b}}{a^3\,b-a^4}}\right)\sqrt{\frac{a^2+\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}$$
$$-2\,\mathrm{atanh}\left(\frac{8\,a^2\,b\tan(x)\,\sqrt{\frac{a^2}{16\,(a^3\,b-a^4)}-\frac{\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}}{2\,a\,b+\frac{2\,a^5\,b}{a^3\,b-a^4}-\frac{2\,a^3\,b\,\sqrt{a^3\,b}}{a^3\,b-a^4}}-\frac{8\,a^6\,b\tan(x)\,\sqrt{\frac{a^2}{16\,(a^3\,b-a^4)}-\frac{\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}}{2\,a^5\,b-2\,a^4\,b^2-\frac{2\,a^8\,b^2}{a^3\,b-a^4}+\frac{2\,a^9\,b}{a^3\,b-a^4}+\frac{2\,a^6\,b^2\,\sqrt{a^3\,b}}{a^3\,b-a^4}-\frac{2\,a^7\,b\,\sqrt{a^3\,b}}{a^3\,b-a^4}}+\frac{8\,a^4\,b\tan(x)\,\sqrt{\frac{a^2}{16\,(a^3\,b-a^4)}-\frac{\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}\,\sqrt{a^3\,b}}{2\,a^5\,b-2\,a^4\,b^2-\frac{2\,a^8\,b^2}{a^3\,b-a^4}+\frac{2\,a^9\,b}{a^3\,b-a^4}+\frac{2\,a^6\,b^2\,\sqrt{a^3\,b}}{a^3\,b-a^4}-\frac{2\,a^7\,b\,\sqrt{a^3\,b}}{a^3\,b-a^4}}\right)\sqrt{\frac{a^2-\sqrt{a^3\,b}}{16\,(a^3\,b-a^4)}}$$

input
```
int(1/(a - b*cos(x)^4),x)
```

output
```
2*atanh((8*a^6*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^3*b)^(1/2)/(16*(a^3*b
 - a^4)))^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9
*b)/(a^3*b - a^4) - (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^
3*b)^(1/2))/(a^3*b - a^4)) - (8*a^2*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^
3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b + (2*a^5*b)/(a^3*b - a^4) + (
2*a^3*b*(a^3*b)^(1/2))/(a^3*b - a^4)) + (8*a^4*b*tan(x)*(a^2/(16*(a^3*b -
a^4)) + (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b -
2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) - (2*a^6*b
^2*(a^3*b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4)))*
((a^2 + (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) - 2*atanh((8*a^2*b*tan(x)
*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b
 + (2*a^5*b)/(a^3*b - a^4) - (2*a^3*b*(a^3*b)^(1/2))/(a^3*b - a^4)) - (8*a
^6*b*tan(x)*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1
/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b -
 a^4) + (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) - (2*a^7*b*(a^3*b)^(1/2))/
(a^3*b - a^4)) + (8*a^4*b*tan(x)*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(
16*(a^3*b - a^4)))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)
/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) + (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*
b - a^4) - (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4)))*((a^2 - (a^3*b)^(1/2))/
(16*(a^3*b - a^4)))^(1/2)
```

3.71. $\int \frac{1}{a-b\cos^4(x)}\,dx$

3.72 $\int \frac{1}{1+\cos^4(x)}\,dx$

3.72.1 Optimal result

Integrand size = 8, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{1+\cos^4(x)}\,dx &= \frac{x}{2\sqrt{-1+\sqrt{2}}}+\frac{\arctan\left(\frac{\left(-2+\sqrt{2}\right)\cos(x)\sin(x)+\sqrt{-1+\sqrt{2}}(1-2\sin^2(x))}{2+\sqrt{1+\sqrt{2}}+2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)+\left(-2+\sqrt{2}\right)\sin^2(x)}\right)}{4\sqrt{-1+\sqrt{2}}}\\
&+\frac{\arctan\left(\frac{\left(-2+\sqrt{2}\right)\cos(x)\sin(x)+\sqrt{-1+\sqrt{2}}(-1+2\sin^2(x))}{2+\sqrt{1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)+\left(-2+\sqrt{2}\right)\sin^2(x)}\right)}{4\sqrt{-1+\sqrt{2}}}\\
&+\frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\cot(x)+2\cot^2(x)\right)\\
&-\frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left(1+\sqrt{2\left(-1+\sqrt{2}\right)}\cot(x)+\sqrt{2}\cot^2(x)\right)\end{aligned}$$

output
```
1/2*x/(2^(1/2)-1)^(1/2)+1/4*arctan((cos(x)*sin(x)*(-2+2^(1/2))+(-1+2*sin(x
)^2)*(2^(1/2)-1)^(1/2))/(2+sin(x)^2*(-2+2^(1/2))-2*cos(x)*sin(x)*(2^(1/2)-
1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)+1/4*arctan((cos(x)*sin(x)*(
-2+2^(1/2))+(1-2*sin(x)^2)*(2^(1/2)-1)^(1/2))/(2+sin(x)^2*(-2+2^(1/2))+2*c
os(x)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)+1/8*l
n(2*cot(x)^2+2^(1/2)-2*cot(x)*(2^(1/2)-1)^(1/2))*(2^(1/2)-1)^(1/2)-1/8*ln(
1+cot(x)^2*2^(1/2)+cot(x)*(-2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.15

$$\int \frac{1}{1+\cos^4(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{2\sqrt{1-i}}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{2\sqrt{1+i}}$$

input
```
Integrate[(1 + Cos[x]^4)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTan[Tan[x]/Sqrt[1 + I]]/(2
*Sqrt[1 + I])
```

3.72.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3688, 1483, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^4(x)+1}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^4+1}dx$$

$$\Big\downarrow \; 3688$$

$$-\int \frac{\cot^2(x)+1}{2\cot^4(x)+2\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 1483$$

$$-\frac{\int \frac{2\sqrt{-1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\int \frac{\left(2-\sqrt{2}\right)\cot(x)+2\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \; 1142$$

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)-\frac{1}{4}\left(2-\sqrt{2}\right)\int -\frac{2\left(\sqrt{-1+\sqrt{2}}-2\cot(x)\right)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{2\left(2\cot(x)+\sqrt{-1+\sqrt{2}}\right)}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$\big\downarrow$ 27

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\cot(x)+\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$\big\downarrow$ 1083

$$-\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\cot(x)-2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\left(4\cot(x)-2\right.}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$
$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\cot(x)+\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\cot(x)+2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\left(4\cot(x)+2\sqrt{}\right.}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$\big\downarrow$ 217

$$-\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{\arctan\left(\frac{4\cot(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\cot(x)+\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{\arctan\left(\frac{4\cot(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$\big\downarrow$ 1103

3.72. $\int \frac{1}{1+\cos^4(x)}\,dx$

$$-\frac{\frac{\arctan\left(\frac{4\cot(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}-\frac{1}{4}(2-\sqrt{2})\log\left(2\cot^2(x)-2\sqrt{\sqrt{2}-1}\cot(x)+\sqrt{2}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\frac{\arctan\left(\frac{4\cot(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}+\frac{1}{4}(2-\sqrt{2})\log\left(\sqrt{2}\cot^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\cot(x)+1\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

input
```
Int[(1 + Cos[x]^4)^(-1),x]
```

output
```
-1/2*(ArcTan[(-2*Sqrt[-1 + Sqrt[2]] + 4*Cot[x])/(2*Sqrt[1 + Sqrt[2]])]/Sqr
t[2] - ((2 - Sqrt[2])*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Cot[x] + 2*Cot[x]
^2])/4)/Sqrt[2*(-1 + Sqrt[2])] - (ArcTan[(2*Sqrt[-1 + Sqrt[2]] + 4*Cot[x])
/(2*Sqrt[1 + Sqrt[2]])]/Sqrt[2] + ((2 - Sqrt[2])*Log[1 + Sqrt[2*(-1 + Sqrt
[2])]*Cot[x] + Sqrt[2]*Cot[x]^2])/4)/(2*Sqrt[2*(-1 + Sqrt[2])])
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.72.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.43

method	result
risch	$\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{-2+2i}-\sqrt{-2+2i}+1-2i\right)}{8}-\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2+2i}+\sqrt{-2+2i}+1-2i\right)}{8}+\frac{\sqrt{-2-2i}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{-2-2i}+ \right.}{8}$
default	$-\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{2\left(-1-\sqrt{2}\right)\arctan\left(\frac{2\tan(x)-\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right)}{\sqrt{2\sqrt{2}+2}}\right)}{8}-\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)+\tan \right.}{2}\right.}{}$

input
```
int(1/(1+cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-2+2*I)^(1/2)*ln(exp(2*I*x)-I*(-2+2*I)^(1/2)-(-2+2*I)^(1/2)+1-2*I)-1/
8*(-2+2*I)^(1/2)*ln(exp(2*I*x)+I*(-2+2*I)^(1/2)+(-2+2*I)^(1/2)+1-2*I)+1/8*
(-2-2*I)^(1/2)*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+1+2*I+(-2-2*I)^(1/2))-1/8*(-
2-2*I)^(1/2)*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)+1+2*I-(-2-2*I)^(1/2))
```

3.72.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.40

$$\int \frac{1}{1+\cos^4(x)}\,dx = \frac{1}{16}\sqrt{2}\sqrt{i-1}\log\left(-(i-1)\sqrt{2}\sqrt{i-1}\cos(x)\sin(x)+(2i-1)\cos(x)^2-i\right) - \frac{1}{16}\sqrt{2}\sqrt{i-1}\log\left((i-1)\sqrt{2}\sqrt{i-1}\cos(x)\sin(x)+(2i-1)\cos(x)^2-i\right) - \frac{1}{16}\sqrt{2}\sqrt{-i-1}\log\left((i+1)\sqrt{2}\sqrt{-i-1}\cos(x)\sin(x)+(2i+1)\cos(x)^2-i\right) + \frac{1}{16}\sqrt{2}\sqrt{-i-1}\log\left(-(i+1)\sqrt{2}\sqrt{-i-1}\cos(x)\sin(x)+(2i+1)\cos(x)^2-i\right)$$

input
```
integrate(1/(1+cos(x)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*sqrt(I - 1)*log(-(I - 1)*sqrt(2)*sqrt(I - 1)*cos(x)*sin(x) +
(2*I - 1)*cos(x)^2 - I) - 1/16*sqrt(2)*sqrt(I - 1)*log((I - 1)*sqrt(2)*sqr
t(I - 1)*cos(x)*sin(x) + (2*I - 1)*cos(x)^2 - I) - 1/16*sqrt(2)*sqrt(-I -
1)*log((I + 1)*sqrt(2)*sqrt(-I - 1)*cos(x)*sin(x) + (2*I + 1)*cos(x)^2 - I
) + 1/16*sqrt(2)*sqrt(-I - 1)*log(-(I + 1)*sqrt(2)*sqrt(-I - 1)*cos(x)*sin
(x) + (2*I + 1)*cos(x)^2 - I)
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**4),x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \frac{1}{1+\cos^4(x)}\,dx = \int \frac{1}{\cos(x)^4+1}\,dx$$

input
```
integrate(1/(1+cos(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(cos(x)^4 + 1), x)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.58

$$\begin{aligned}\int \frac{1}{1+\cos^4(x)}\,dx &= \frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1} \\ &\quad+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1} \\ &\quad-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right) \\ &\quad+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)\end{aligned}$$

input
```
integrate(1/(1+cos(x)^4),x, algorithm="giac")
```

output
```
1/4*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2)
 + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x/pi +
1/2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sq
rt(2) + 2)))*sqrt(sqrt(2) + 1) - 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^2 + 2^(1
/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1/8*sqrt(sqrt(2) - 1)*log(tan(x
)^2 - 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2))
```

3.72.9 Mupad [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.73

$$\int \frac{1}{1+\cos^4(x)}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1} + \frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1} - \frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)$$

input
```
int(1/(cos(x)^4 + 1),x)
```

output
```
atanh((4*2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/6
4)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) + (4*2^(1/2)*tan(x)*(2^(1/2)/64
- 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) -
 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2)) - atanh
((4*2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1
/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1) - (4*2^(1/2)*tan(x)*(2^(1/2)/64 - 1/6
4)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1))*
(2*(- 2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))
```

3.73 $\int \frac{1}{1-\cos^4(x)}\,dx$

3.73.1 Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{x}{2\sqrt{2}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{2\sqrt{2}} - \frac{\cot(x)}{2}$$

output
```
-1/2*cot(x)+1/4*x*2^(1/2)-1/4*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2
^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right) - 2\cot(x)\right)$$

input
```
Integrate[(1 - Cos[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/4
```

3.73.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3688, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^4(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^4}dx$$
$$\Big\downarrow \ 3688$$
$$-\int \frac{\cot^2(x)+1}{2\cot^2(x)+1}d\cot(x)$$
$$\Big\downarrow \ 299$$
$$-\frac{1}{2}\int \frac{1}{2\cot^2(x)+1}d\cot(x)-\frac{\cot(x)}{2}$$
$$\Big\downarrow \ 216$$
$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{2\sqrt{2}}-\frac{\cot(x)}{2}$$

input
```
Int[(1 - Cos[x]^4)^(-1),x]
```

output
```
-1/2*ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2] - Cot[x]/2
```

3.73.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.73.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{1}{2\tan(x)}+\frac{\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)\sqrt{2}}{4}$	21
risch	$-\frac{i}{e^{2ix}-1}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{8}$	52

input
```
int(1/(1-cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/tan(x)+1/4*arctan(1/2*2^(1/2)*tan(x))*2^(1/2)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\cos^4(x)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)\sin(x)+4\cos(x)}{8\sin(x)}$$

input
```
integrate(1/(1-cos(x)^4),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))*s
in(x) + 4*cos(x))/sin(x)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)}{4} - \frac{1}{4\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**4),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4 + tan(x/2)/4
 - 1/(4*tan(x/2))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - \frac{1}{2\tan(x)}$$

input
```
integrate(1/(1-cos(x)^4),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/2/tan(x)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-\frac{1}{2\tan(x)}$$

input
```
integrate(1/(1-cos(x)^4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) - 1/2/tan(x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}-\frac{1}{2\tan(x)}$$

input
```
int(-1/(cos(x)^4 - 1),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tan(x))/2))/4 - 1/(2*tan(x))
```

3.74 $\int \frac{1}{a+b\cos^5(x)}\,dx$

3.74.1 Optimal result

Integrand size = 10, antiderivative size = 494

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

output
```
2/5*arctan((a^(1/5)-b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+b^(1/5))^(1/2))/a^(
4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(
-1)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2))/a^
(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2
)+2/5*arctan((a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(
2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(
-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)*tan
(1/2*x)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^
(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(
4/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)
/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

3.74.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.26

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \frac{8}{5}\text{RootSum}\left[b+5b\#1^2+10b\#1^4+32a\#1^5+10b\#1^6+5b\#1^8 + b\#1^{10}\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3 - i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b+4b\#1^2+16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a + b*Cos[x]^5)^(-1),x]
```

output
```
(8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b
*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 +
#1^2]*#1^3)/(b + 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])
/5
```

3.74.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^5(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}-\frac{1}{5a^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{b}\cos(x)-\sqrt[5]{a}\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)\right)}-\frac{}{5a^{4/5}\left(\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

input
```
Int[(a + b*Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a^(1/5) - b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + b^(1/5)]])/(5*
a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt
[a^(1/5) + (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)
]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5
)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt
[a^(1/5) + (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1
/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^
(3/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*S
qrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2
*ArcTan[(Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^
(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5
) + (-1)^(4/5)*b^(1/5)])
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.74.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{10}+(5a+5b)\_Z^{8}+(10a-10b)\_Z^{6}+(10a+10b)\_Z^{4}+(5a-5b)\_Z^{2}+a+b\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+\right.}{\_R^{9}a-\_R^{9}b+4\_R^{7}a+4\_R^{7}b+6\_R^{5}}\right.}{5}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(-\right.\right.$

input
```
int(1/(a+b*cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a-_R^9*b+4*_R^7*a+4*_R^7*b+6*_
R^5*a-6*_R^5*b+4*_R^3*a+4*_R^3*b+_R*a-_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a
-b)*_Z^10+(5*a+5*b)*_Z^8+(10*a-10*b)*_Z^6+(10*a+10*b)*_Z^4+(5*a-5*b)*_Z^2+
a+b))
```

3.74.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*cos(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.74.6 Sympy [F]

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \int \frac{1}{a+b\cos^5(x)}\,dx$$

input
```
integrate(1/(a+b*cos(x)**5),x)
```

output
```
Integral(1/(a + b*cos(x)**5), x)
```

3.74.7 Maxima [F]

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \int \frac{1}{b\cos(x)^5+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^5),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^5 + a), x)
```

3.74.8 Giac [F]

$$\int \frac{1}{a+b\cos^5(x)}\,dx=\int \frac{1}{b\cos(x)^5+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^5),x, algorithm="giac")
```

output
```
integrate(1/(b*cos(x)^5 + a), x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 1520, normalized size of antiderivative = 3.08

$$\int \frac{1}{a+b\cos^5(x)}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*cos(x)^5),x)
```

output
```
symsum(log(-(10995116277760*b^7*(a - b)*(7*cot(x/2) - 56*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4
*d^4 - 125*a^2*d^2 - 1, d, k)*a + root(9765625*a^8*b^2*d^10 - 9765625*a^10
*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)*b - 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 - 225
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1562
50*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 - 3875000*root(97
65625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6
- 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 - 25000000*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4
*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 735*root(9765625*a^8*b^2*d^10 - 9765
625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)^2*a^2*cot(x/2) + 28875*root(9765625*a^8*b^2*d^10 - 9765625*
a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2
- 1, d, k)^4*a^4*cot(x/2) + 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1
, d, k)^6*a^6*cot(x/2) + 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*
d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^8*a^8*cot(x/2) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1...
```

3.75 $\int \frac{1}{a+b\cos^6(x)}\,dx$

3.75.1 Optimal result

Integrand size = 10, antiderivative size = 171

$$\int \frac{1}{a+b\cos^6(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
-1/3*arctan(cot(x)*(a^(1/3)+b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/
3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/a^(1/6))/a^
(5/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)+(-1)^(
2/3)*b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \frac{8}{3}\text{RootSum}\Bigg[b+6b\#1+15b\#1^2+64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5 + b\#1^6\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{b+5b\#1+32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\Bigg]$$

input
```
Integrate[(a + b*Cos[x]^6)^(-1),x]
```

output
```
(8*RootSum[b + 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2
*Cos[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 + 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3
 + 5*b*#1^4 + b*#1^5) & ])/3
```

3.75.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{\int \frac{1}{\left(\frac{\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{3a}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a + b*Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Cot[x])/a^(1/6)]/(a^(5/6)*Sqrt[a^(1/3
) + b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)
]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) +
(-1)^(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*
b^(1/3)])
```

3.75.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.75.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 a+3\_Z^4 a+3\_Z^2 a+a+b\right)} \frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5+2\_R^3+\_R}}{6a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6+46656a^5 b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{15552ia^6}{b}-15552ia^5\right)\_R^5+\right.$

input
```
int(1/(a+b*cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+
3*_Z^4*a+3*_Z^2*a+a+b))
```

3.75.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 15483, normalized size of antiderivative = 90.54

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

3.75.6 Sympy [F]

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \int \frac{1}{a+b\cos^6(x)}\,dx$$

input
```
integrate(1/(a+b*cos(x)**6),x)
```

output
```
Integral(1/(a + b*cos(x)**6), x)
```

3.75.7 Maxima [F]

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \int \frac{1}{b\cos(x)^6+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^6),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^6 + a), x)
```

3.75.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)^6),x, algorithm="giac")
```

output
```
Timed out
```

3.75.9 Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \sum_{k=1}^{6} \ln\Big(\text{root}\big(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\big)^2\,a^3\,b^3\,\Big(\text{root}\big(46656\,a^5\,b\,d^6+466\ldots + 1\Big)\,\big(\text{root}\big(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\big)\,a\tan(x)\,6 - 1\big)\,36\Big)\,\text{root}\big(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\big)$$

input
```
int(1/(a + b*cos(x)^6),x)
```

output
```
symsum(log(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^
2*d^2 + 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888
*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 + 466
56*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(466
56*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k, 1
, 6)
```

3.76 $\int \frac{1}{a+b\cos^8(x)}\,dx$

3.76.1 Optimal result

Integrand size = 10, antiderivative size = 245

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

output
```
1/4*arctan(cot(x)*((-a)^(1/4)-b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctan(cot(x)*((-a)^(1/4)-I*b^(1/4))^(1/2)/(-a)^(
1/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)+1/4*arctan(cot(x)*((-a)^(1/4
)+I*b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)+1/4
*arctan(cot(x)*((-a)^(1/4)+b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/
4)+b^(1/4))^(1/2)
```

3.76.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+b\cos^8(x)}\,dx$$
$$= 8\text{RootSum}\Big[b+8b\#1+28b\#1^2+56b\#1^3+256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7 +b\#1^8\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{b+7b\#1+21b\#1^2+128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\Big]$$

input
```
Integrate[(a + b*Cos[x]^8)^(-1),x]
```

output
```
8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 + 56
*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
 #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^2
 + 128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

3.76.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^8(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^8}dx$$
$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \textbf{3660}$$

$$-\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}a}{(-a)^{5/4}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{(-a)^{5/8}\arctan\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\cot(x)}{(-a)^{5/8}}\right)}{4a\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

input
```
Int[(a + b*Cos[x]^8)^(-1),x]
```

output
```
-1/4*((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Cot[x])/(-a)^(1/8)])
/(a*Sqrt[(-a)^(1/4) - I*b^(1/4)]) - ((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) +
I*b^(1/4)]*Cot[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ((-a)
^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) + b^(1/4)]*Cot[x])/(-a)^(1/8)])/(4*a*Sqrt[(
-a)^(1/4) + b^(1/4)]) - ((-a)^(5/8)*ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*C
ot[x])/(-a)^(5/8)])/(4*a*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

3.76. $\int \frac{1}{a+b\cos^8(x)}\,dx$

3.76.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.76.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.31

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8 a+4\_Z^6 a+6\_Z^4 a+4\_Z^2 a+a+b\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$
risch	$\sum_{\_R=\text{RootOf}\left(1+(16777216a^8+16777216b\,a^7)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)} \_R \ln\left(e^{2ix}+\left(\frac{4194304ia^8}{b}+\right.\right.$

input
```
int(1/(a+b*cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=
RootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a+b))
```

3.76.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665467 vs. 2(165) = 330.

Time = 6.13 (sec) , antiderivative size = 665467, normalized size of antiderivative = 2716.19

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

3.76.6 Sympy [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \int \frac{1}{a+b\cos^8{(x)}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**8),x)
```

output
```
Integral(1/(a + b*cos(x)**8), x)
```

3.76.7 Maxima [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \int \frac{1}{b\cos{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^8 + a), x)
```

3.76.8 Giac [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx=\int \frac{1}{b\cos{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^8),x, algorithm="giac")
```

output
```
integrate(1/(b*cos(x)^8 + a), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\cos^8(x)}\,dx$$
$$=\sum_{k=1}^{8}\ln\Big(\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^4\,a^5\,b^5\,($$
$$+1\Big)\,\big(\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)\,a\tan(x)\,8$$
$$-1\big)\,4096\Big)\,\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4$$
$$+256\,a^2\,d^2+1,d,k\big)$$

input
```
int(1/(a + b*cos(x)^8),x)
```

output
```
symsum(log(4096*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d
^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^4*a^5*b^5*(64*root(16777216*a^
7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2
 + 1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 104
8576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(
16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 +
256*a^2*d^2 + 1, d, k), k, 1, 8)
```

3.77 $\int \frac{1}{a-b\cos^5(x)}\,dx$

3.77.1 Optimal result

Integrand size = 11, antiderivative size = 494

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}$$
$$+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}$$
$$+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}$$
$$+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

output
```
2/5*arctan((a^(1/5)+b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-b^(1/5))^(1/2))/a^(
4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(
-1)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2))/a^
(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2
)+2/5*arctan((a^(1/5)+(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(
2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(
-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2)*tan
(1/2*x)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^
(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(
4/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)
/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

3.77.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.26

$$\int \frac{1}{a-b\cos^5(x)}\,dx = -\frac{8}{5}\text{RootSum}\Bigg[b+5b\#1^2+10b\#1^4-32a\#1^5+10b\#1^6+5b\#1^8 + b\#1^{10}\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b+4b\#1^2-16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\Bigg]$$

input
```
Integrate[(a - b*Cos[x]^5)^(-1),x]
```

output
```
(-8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 - 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 +
b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 +
 #1^2]*#1^3)/(b + 4*b*#1^2 - 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ]
)/5
```

3.77.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^5(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{a}+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

input
```
Int[(a - b*Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a^(1/5) + b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - b^(1/5)]])/(5*
a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt
[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(1/5)*b^(1/5)
]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5
)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt
[a^(1/5) - (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1
/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^
(3/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*S
qrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2
*ArcTan[(Sqrt[a^(1/5) + (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^
(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5
) + (-1)^(4/5)*b^(1/5)])
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.77.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left((a+b)\_Z^{10}+(5a-5b)\_Z^{8}+(10a+10b)\_Z^{6}+(10a-10b)\_Z^{4}+(5a+5b)\_Z^{2}+a-b\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\right.}{\_R^{9}a+\_R^{9}b+4\_R^{7}a-4\_R^{7}b+6\_R^{5}}\right.}{5}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(e^{ix}+\left(1\right.\right.$

input
```
int(1/(a-b*cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+_R^9*b+4*_R^7*a-4*_R^7*b+6*_
R^5*a+6*_R^5*b+4*_R^3*a-4*_R^3*b+_R*a+_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a
+b)*_Z^10+(5*a-5*b)*_Z^8+(10*a+10*b)*_Z^6+(10*a-10*b)*_Z^4+(5*a+5*b)*_Z^2+
a-b))
```

3.77.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-b*cos(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.77.6 Sympy [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \int \frac{1}{a-b\cos^5{(x)}}\,dx$$

input
```
integrate(1/(a-b*cos(x)**5),x)
```

output
```
Integral(1/(a - b*cos(x)**5), x)
```

3.77.7 Maxima [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \int -\frac{1}{b\cos{(x)}^5-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^5),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^5 - a), x)
```

3.77.8 Giac [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx=\int -\frac{1}{b\cos{(x)}^5-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^5),x, algorithm="giac")
```

output
```
integrate(-1/(b*cos(x)^5 - a), x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 1518, normalized size of antiderivative = 3.07

$$\int \frac{1}{a-b\cos^5(x)}\,dx=\text{Too large to display}$$

input
```
int(1/(a - b*cos(x)^5),x)
```

output
```
symsum(log(-(10995116277760*b^7*(a + b)*(56*root(9765625*a^8*b^2*d^10 - 97
65625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^
2*d^2 - 1, d, k)*a - 7*cot(x/2) + root(9765625*a^8*b^2*d^10 - 9765625*a^10
*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)*b + 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 + 225
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1562
50*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 3875000*root(97
65625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6
- 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 25000000*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4
*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 - 735*root(9765625*a^8*b^2*d^10 - 9765
625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)^2*a^2*cot(x/2) - 28875*root(9765625*a^8*b^2*d^10 - 9765625*
a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2
- 1, d, k)^4*a^4*cot(x/2) - 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1
, d, k)^6*a^6*cot(x/2) - 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*
d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^8*a^8*cot(x/2) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1...
```

3.78 $\int \frac{1}{a-b\cos^6(x)}\,dx$

3.78.1 Optimal result

Integrand size = 11, antiderivative size = 175

$$\int \frac{1}{a-b\cos^6(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
-1/3*arctan(cot(x)*(a^(1/3)-b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/
3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)/a^(1/6))/a^
(5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)-(-1)^(
2/3)*b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.83

$$\int \frac{1}{a-b\cos^6(x)}\,dx$$
$$= -\frac{8}{3}\text{RootSum}\left[b+6b\#1+15b\#1^2-64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5\right.$$
$$\left.+b\#1^6\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{b+5b\#1-32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\right]$$

input
```
Integrate[(a - b*Cos[x]^6)^(-1),x]
```

output
```
(-8*RootSum[b + 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6
*b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 -
2*Cos[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 - 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^
3 + 5*b*#1^4 + b*#1^5) & ])/3
```

3.78.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^6(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^6}dx$$
$$\Big\downarrow \; 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}$$

$\downarrow$ 3042

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}$$

$\downarrow$ 3660

$$-\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\cot^2(x)+1}d\cot(x)}{3a}$$

$\downarrow$ 216

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Cot[x])/a^(1/6)]/(a^(5/6)*Sqrt[a^(1/3
) - b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)
]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) -
(-1)^(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*
b^(1/3)])
```

3.78.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.89 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.35

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6a+3\_Z^4a+3\_Z^2a+a-b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5+2\_R^3+\_R}}{6a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6-46656a^5b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{15552ia^6}{b}-15552ia^5\right)\_R^5+\left(\right.\right.$

input
```
int(1/(a-b*cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+
3*_Z^4*a+3*_Z^2*a+a-b))
```

3.78.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 16679, normalized size of antiderivative = 95.31

$$\int \frac{1}{a - b\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

3.78.6 Sympy [F]

$$\int \frac{1}{a - b\cos^6(x)}\,dx = \int \frac{1}{a - b\cos^6(x)}\,dx$$

input
```
integrate(1/(a-b*cos(x)**6),x)
```

output
```
Integral(1/(a - b*cos(x)**6), x)
```

3.78.7 Maxima [F]

$$\int \frac{1}{a - b\cos^6(x)}\,dx = \int -\frac{1}{b\cos(x)^6 - a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^6),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^6 - a), x)
```

3.78.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cos(x)^6),x, algorithm="giac")
```

output
```
Timed out
```

3.78.9 Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \sum_{k=1}^{6} \ln\Big(-\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\big)^2\,a^3\,b^3\,\Big(\mathrm{root}\big(46656\,a^5\,b\,d^6-4 \\ +1\Big)\,\big(\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\big)\,a\tan(x)\,6 \\ -1\big)\,36\Big)\,\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\big)$$

input
```
int(1/(a - b*cos(x)^6),x)
```

output
```
symsum(log(-36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a
^2*d^2 - 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 388
8*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 - 46
656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root(46
656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k,
1, 6)
```

3.79 $\int \frac{1}{a-b\cos^8(x)}\,dx$

3.79.1 Optimal result

Integrand size = 11, antiderivative size = 213

$$\int \frac{1}{a-b\cos^8(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

output
```
-1/4*arctan(cot(x)*(a^(1/4)-b^(1/4))^(1/2)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/
4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)-I*b^(1/4))^(1/2)/a^(1/8))/a^(7/8)/(a^
(1/4)-I*b^(1/4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)+I*b^(1/4))^(1/2)/a^(1/8)
)/a^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)+b^(1/4))^(1
/2)/a^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

3.79.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81

$$\int \frac{1}{a-b\cos^8(x)}\,dx$$
$$=-8\text{RootSum}\Big[b+8b\#1+28b\#1^2+56b\#1^3-256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7+b\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{b+7b\#1+21b\#1^2-128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\Big]$$

input
```
Integrate[(a - b*Cos[x]^8)^(-1),x]
```

output
```
-8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 + 5
6*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x]
- #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^
2 - 128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) &
]
```

3.79.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^8(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^8}dx$$
$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{\int \frac{1}{\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

input
```
Int[(a - b*Cos[x]^8)^(-1),x]
```

output
```
-1/4*ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Cot[x])/a^(1/8)]/(a^(7/8)*Sqrt[a^(1/4
) - b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7
/8)*Sqrt[a^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Cot[x])
/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + b
^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

3.79. $\int \frac{1}{a-b\cos^8(x)}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.37

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8 a+4\_Z^6 a+6\_Z^4 a+4\_Z^2 a+a-b\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$
risch	$\sum_{\_R=\text{RootOf}\left(1+(16777216a^8-16777216b\,a^7)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)} \_R\ln\left(e^{2ix}+\left(-\frac{4194304ia^8}{b}\right.\right.$

input
```
int(1/(a-b*cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=
RootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a-b))
```

3.79.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643291 vs. 2(133) = 266.

Time = 6.08 (sec) , antiderivative size = 643291, normalized size of antiderivative = 3020.15

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

3.79.6 Sympy [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \int \frac{1}{a-b\cos^8(x)}\,dx$$

input
```
integrate(1/(a-b*cos(x)**8),x)
```

output
```
Integral(1/(a - b*cos(x)**8), x)
```

3.79.7 Maxima [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \int -\frac{1}{b\cos(x)^8-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^8),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^8 - a), x)
```

3.79.8 Giac [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \int -\frac{1}{b\cos{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^8),x, algorithm="giac")
```

output
```
integrate(-1/(b*cos(x)^8 - a), x)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int \frac{1}{a-b\cos^8(x)}\,dx$$
$$= \sum_{k=1}^{8} \ln\Big(-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^4\,a^5\,b$$
$$+1\Big)\ \big(\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)\ a\tan(x)\ 8$$
$$-\,1\big)\ 4096\Big)\ \mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4$$
$$-\,256\,a^2\,d^2-1,d,k\big)$$

input
```
int(1/(a - b*cos(x)^8),x)
```

output
```
symsum(log(-4096*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*
d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^4*a^5*b^5*(64*root(16777216*a
^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^
2 - 1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 10
48576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root
(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 -
 256*a^2*d^2 - 1, d, k), k, 1, 8)
```

3.80 $\int \frac{1}{1+\cos^5(x)} \, dx$

3.80.1 Optimal result

Integrand size = 8, antiderivative size = 223

$$\int \frac{1}{1+\cos^5(x)} \, dx = \frac{2\arctan\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\arctan\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}} - \frac{2\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}} - \frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\operatorname{arctanh}\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)} + \frac{\sin(x)}{5(1+\cos(x))}$$

output
```
1/5*sin(x)/(1+cos(x))-2/5*arctanh(tan(1/2*x)/((-1+(-1)^(1/5))/(1+(-1)^(1/5
)))^(1/2))/(-1+(-1)^(2/5))^(1/2)+2/5*arctan(((1-(-1)^(4/5))/(1+(-1)^(4/5))
)^(1/2)*tan(1/2*x))/(1+(-1)^(3/5))^(1/2)-2/5*arctanh(((-1-(-1)^(3/5))/(1-(
-1)^(3/5)))^(1/2)*tan(1/2*x))*((-1-(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)/(1+(-
1)^(3/5))+2/5*arctan(((1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2)*tan(1/2*x))/(1-
(-1)^(4/5))^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{1}{1+\cos^5(x)}\,dx = -\frac{1}{10}\text{RootSum}\Bigg[1-2\#1+8\#1^2-14\#1^3+30\#1^4-14\#1^5+8\#1^6-2\#1^7$$
$$+\#1^8\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)-8\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1+4i\log\left(1-2\right.}{}$$
$$+\frac{1}{5}\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x]^5)^(-1),x]
```

output
```
-1/10*RootSum[1 - 2*#1 + 8*#1^2 - 14*#1^3 + 30*#1^4 - 14*#1^5 + 8*#1^6 - 2
*#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos[x]*#1 +
 #1^2] - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + (4*I)*Log[1 - 2*Cos[x]*#1 + #
1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 - 2*Cos[x]*#
1 + #1^2]*#1^2 - 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 + (40*I)*Log[1 - 2*C
os[x]*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (15*I)*Log[
1 - 2*Cos[x]*#1 + #1^2]*#1^4 - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^5 + (4*I)
*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 -
I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-1 + 8*#1 - 21*#1^2 + 60*#1^3 - 35*#1
^4 + 24*#1^5 - 7*#1^6 + 4*#1^7) & ] + Tan[x/2]/5
```

3.80.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^5(x)+1}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^5+1}dx$$

$\big\downarrow$ 3692

$$\int \left(-\frac{1}{5\left(\sqrt[5]{-1}\cos(x)-1\right)}-\frac{1}{5\left(-(-1)^{2/5}\cos(x)-1\right)}-\frac{1}{5\left((-1)^{3/5}\cos(x)-1\right)}-\frac{1}{5\left(-(-1)^{4/5}\cos(x)-1\right)}-\frac{1}{5(-\right.$$

$\big\downarrow$ 2009

$$\frac{2\arctan\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\arctan\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)}+\frac{\sin(x)}{5(\cos(x)+1)}$$

input
```
Int[(1 + Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[Sqrt[(1 - (-1)^(2/5))/(1 + (-1)^(2/5))]*Tan[x/2]])/(5*Sqrt[1 - (
-1)^(4/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(4/5))/(1 + (-1)^(4/5))]*Tan[x/2]])
/(5*Sqrt[1 + (-1)^(3/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(1/5))/(1
 + (-1)^(1/5)))]])/(5*Sqrt[-1 + (-1)^(2/5)]) - (2*Sqrt[-((1 + (-1)^(3/5))/
(1 - (-1)^(3/5)))]*ArcTanh[Sqrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*Tan[
x/2]])/(5*(1 + (-1)^(3/5))) + Sin[x]/(5*(1 + Cos[x]))
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.80.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.28

method	result
default	$\frac{\tan\left(\frac{x}{2}\right)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^{8}+10\_Z^{4}+1\right)}\frac{\left(5\_R^{6}+5\_R^{4}+5\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}+\_R^{3}}\right)}{50}$
risch	$\frac{2i}{5(\mathrm{e}^{ix}+1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^{8}+156250\_Z^{6}+6250\_Z^{4}+125\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{ix}-2343750i\_R^{7}+234375\right.\right.$

input
```
int(1/(1+cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*tan(1/2*x)+1/50*sum((5*_R^6+5*_R^4+5*_R^2+1)/(_R^7+_R^3)*ln(tan(1/2*x)
-_R),_R=RootOf(5*_Z^8+10*_Z^4+1))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(150) = 300.

Time = 0.40 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.54

$$\int \frac{1}{1+\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+cos(x)^5),x, algorithm="fricas")
```

output
```
1/100*((sqrt(5)*cos(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)
*log(sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(
5) - 5)*sin(x) - 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*cos(x) - 5*(sqrt(5) -
 1)*cos(x) - 20) - (sqrt(5)*cos(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(2*sqrt(5
) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)
*sqrt(2*sqrt(5) - 5)*sin(x) - 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*cos(x) -
 5*(sqrt(5) - 1)*cos(x) - 20) + (sqrt(5)*cos(x) + sqrt(5))*sqrt(-2*sqrt(5)
*sqrt(2*sqrt(5) - 5) - 10)*log(sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(
3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*sin(x) - 5*sqrt(2*sqrt(5) - 5)*(sqrt(5)
 + 3)*cos(x) + 5*(sqrt(5) - 1)*cos(x) + 20) - (sqrt(5)*cos(x) + sqrt(5))*s
qrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(
5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*sin(x) - 5*sqrt(2*sqrt(5
) - 5)*(sqrt(5) + 3)*cos(x) + 5*(sqrt(5) - 1)*cos(x) + 20) - (sqrt(5)*cos(
x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*log(sqrt(2*sqrt(5)
*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) - 5)*sin(x) -
5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5)*cos(x) + 5*(sqrt(5) + 1)*cos(x) - 20)
 + (sqrt(5)*cos(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*lo
g(-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(
5) - 5)*sin(x) - 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5)*cos(x) + 5*(sqrt(5)
+ 1)*cos(x) - 20) - (sqrt(5)*cos(x) + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2*...
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^5(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**5),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{1}{1+\cos^5(x)}\,dx = \int \frac{1}{\cos(x)^5+1}\,dx$$

input
```
integrate(1/(1+cos(x)^5),x, algorithm="maxima")
```

output
```
-1/5*(5*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(-2/5*((cos(7*x) - 4
*cos(6*x) + 15*cos(5*x) - 40*cos(4*x) + 15*cos(3*x) - 4*cos(2*x) + cos(x))
*cos(8*x) + (16*cos(6*x) - 44*cos(5*x) + 110*cos(4*x) - 44*cos(3*x) + 16*c
os(2*x) - 4*cos(x) + 1)*cos(7*x) - 2*cos(7*x)^2 + 4*(44*cos(5*x) - 110*cos
(4*x) + 44*cos(3*x) - 16*cos(2*x) + 4*cos(x) - 1)*cos(6*x) - 32*cos(6*x)^2
 + (1010*cos(4*x) - 420*cos(3*x) + 176*cos(2*x) - 44*cos(x) + 15)*cos(5*x)
 - 210*cos(5*x)^2 + 10*(101*cos(3*x) - 44*cos(2*x) + 11*cos(x) - 4)*cos(4*
x) - 1200*cos(4*x)^2 + (176*cos(2*x) - 44*cos(x) + 15)*cos(3*x) - 210*cos(
3*x)^2 + 4*(4*cos(x) - 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (sin(7*x
) - 4*sin(6*x) + 15*sin(5*x) - 40*sin(4*x) + 15*sin(3*x) - 4*sin(2*x) + si
n(x))*sin(8*x) + 2*(8*sin(6*x) - 22*sin(5*x) + 55*sin(4*x) - 22*sin(3*x) +
 8*sin(2*x) - 2*sin(x))*sin(7*x) - 2*sin(7*x)^2 + 8*(22*sin(5*x) - 55*sin(
4*x) + 22*sin(3*x) - 8*sin(2*x) + 2*sin(x))*sin(6*x) - 32*sin(6*x)^2 + 2*(
505*sin(4*x) - 210*sin(3*x) + 88*sin(2*x) - 22*sin(x))*sin(5*x) - 210*sin(
5*x)^2 + 10*(101*sin(3*x) - 44*sin(2*x) + 11*sin(x))*sin(4*x) - 1200*sin(4
*x)^2 + 44*(4*sin(2*x) - sin(x))*sin(3*x) - 210*sin(3*x)^2 - 32*sin(2*x)^2
 + 16*sin(2*x)*sin(x) - 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) - 8*cos(6*x) +
 14*cos(5*x) - 30*cos(4*x) + 14*cos(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(
8*x) - cos(8*x)^2 + 4*(8*cos(6*x) - 14*cos(5*x) + 30*cos(4*x) - 14*cos(3*x
) + 8*cos(2*x) - 2*cos(x) + 1)*cos(7*x) - 4*cos(7*x)^2 + 16*(14*cos(5*x...
```

3.80.8 Giac [F]

$$\int \frac{1}{1+\cos^5(x)}\,dx = \int \frac{1}{\cos(x)^5+1}\,dx$$

input
```
integrate(1/(1+cos(x)^5),x, algorithm="giac")
```

output
```
sage0*x
```

3.80.9 Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.40

$$\int \frac{1}{1+\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(x)^5 + 1),x)
```

output
```
tan(x/2)/5 + 2*atanh((603979776*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))
/1220703125 - (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5
- 1)^(1/2))/1220703125 - 301989888/1220703125)) + (268435456*5^(1/2)*tan(x
/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432
*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/122
0703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 301989888/12
20703125)))*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((603
979776*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*
((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5^
(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + 30
1989888/1220703125)) + (268435456*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^
(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2))/1220703125 + (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(
1/2))/5 - 1)^(1/2))/1220703125 + 301989888/1220703125)))*((- (2*5^(1/2))/5
 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((603979776*tan(x/2)*(- ((2*5^(1/2))
/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*((2*5^(1/2))
/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/1220703125 - (67108864*((2
*5^(1/2))/5 - 1)^(1/2))/1220703125 + 301989888/1220703125)) - (268435456*5
^(1/2)*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(2441406...
```

3.81 $\int \frac{1}{1+\cos^6(x)}\,dx$

3.81.1 Optimal result

Integrand size = 8, antiderivative size = 83

$$\int \frac{1}{1+\cos^6(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

output
```
1/6*arctan(1/2*2^(1/2)*tan(x))*2^(1/2)+1/3*arctan(tan(x)/(1-(-1)^(1/3))^(1
/2))/(1-(-1)^(1/3))^(1/2)+1/3*arctan(tan(x)/(1+(-1)^(2/3))^(1/2))/(1+(-1)^
(2/3))^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{1}{1+\cos^6(x)}\,dx=\frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)+2\sqrt{3}\arctan\left(\frac{1+2\tan(x)}{\sqrt{3}}\right)+\log(2-\sin(2x))-\log(2+\sin(2x))\right)$$

input
```
Integrate[(1 + Cos[x]^6)^(-1),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Tan[x]/Sqrt[
2]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + Log[2 - Sin[2*x]] - Log[2
 + Sin[2*x]])/12
```

3.81.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^6(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^6+1}dx$$

$$\Big\downarrow \text{ 3690}$$

$$\frac{1}{3}\int \frac{1}{\cos^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\cos^2(x)}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\cos^2(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{ 3660}$$

$$-\frac{1}{3}\int \frac{1}{2\cot^2(x)+1}d\cot(x)-\frac{1}{3}\int \frac{1}{\left(1-\sqrt[3]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{3}\int \frac{1}{\left(1+(-1)^{2/3}\right)\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{3\sqrt{2}}-\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

input
```
Int[(1 + Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2] - ArcTan[Sqrt[1 - (-1)^(1/3)]*Cot[x]]/
(3*Sqrt[1 - (-1)^(1/3)]) - ArcTan[Sqrt[1 + (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 +
 (-1)^(2/3)])
```

3.81.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.81.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result
default	$\frac{\ln(\tan^2(x)-\tan(x)+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(\tan^2(x)+\tan(x)+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}+\arctan\left(\sqrt{2}\right.$
risch	$\frac{i\sqrt{2}\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{12}-\frac{i\sqrt{2}\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{12}-\frac{\ln\left(e^{2ix}+2i+i\sqrt{3}\right)}{12}+\frac{i\ln\left(e^{2ix}+2i+i\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(e^{2ix}+2i-i\sqrt{3}\right)}{12}-i\ln$

input
```
int(1/(1+cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))-1/
12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1)*3^(1/2))+1/6*
arctan(1/2*2^(1/2)*tan(x))*2^(1/2)
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) - \frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right) - \frac{1}{24}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right) + \frac{1}{24}\log\left(-\cos(x)^4+\cos(x)^2-2\cos(x)\sin(x)+1\right)$$

input
```
integrate(1/(1+cos(x)^6),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
1)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x
)^2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)
*sin(x))) - 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/24*lo
g(-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**6),x)
```

output
```
Timed out
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right) + \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - \frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right) + \frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)$$

input
```
integrate(1/(1+cos(x)^6),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/12*log(
tan(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(58) = 116.

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.23

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)+\cos(2x)-2\sin(2x)+1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)-\sin(2x)+2}\right)\right) + \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right) + \frac{1}{6}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right) - \frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right) + \frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)$$

input
```
integrate(1/(1+cos(x)^6),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x
+ arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x)
 + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt
(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) - 1
/12*log(tan(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{6} + \mathrm{atan}\left(\frac{\sqrt{3}\tan(x)}{2} + \frac{\tan(x)\ 1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right) - \mathrm{atan}\left(-\frac{\sqrt{3}\tan(x)}{2} + \frac{\tan(x)\ 1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right) + \frac{(x - \mathrm{atan}(\tan(x)))\left(\frac{\pi\sqrt{2}}{6} + \pi\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right) + \pi\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right)\right)}{\pi}$$

input
```
int(1/(cos(x)^6 + 1),x)
```

output
```
atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) - atan((tan(x)
*1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) + (2^(1/2)*atan((2^(1/2)*t
an(x))/2))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6)
 + pi*(3^(1/2)/6 + 1i/6)))/pi
```

3.82 $\int \frac{1}{1+\cos^8(x)}\,dx$

3.82.1 Optimal result

Integrand size = 8, antiderivative size = 129

$$\int \frac{1}{1+\cos^8(x)}\,dx = -\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

output

```
-1/4*arctan(cot(x)*(1-(-1)^(1/4))^(1/2))/(1-(-1)^(1/4))^(1/2)-1/4*arctan(c
ot(x)*(1+(-1)^(1/4))^(1/2))/(1+(-1)^(1/4))^(1/2)-1/4*arctan(cot(x)*(1-(-1)
^(3/4))^(1/2))/(1-(-1)^(3/4))^(1/2)-1/4*arctan(cot(x)*(1+(-1)^(3/4))^(1/2)
)/(1+(-1)^(3/4))^(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+\cos^8(x)}\,dx$$
$$= 8\text{RootSum}\left[1+8\#1+28\#1^2+56\#1^3+326\#1^4+56\#1^5+28\#1^6+8\#1^7 +\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{1+7\#1+21\#1^2+163\#1^3+35\#1^4+21\#1^5+7\#1^6+\#1^7}\&\right]$$

input
```
Integrate[(1 + Cos[x]^8)^(-1),x]
```

output
```
8*RootSum[1 + 8*#1 + 28*#1^2 + 56*#1^3 + 326*#1^4 + 56*#1^5 + 28*#1^6 + 8*
#1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos
[2*x]*#1 + #1^2]*#1^3)/(1 + 7*#1 + 21*#1^2 + 163*#1^3 + 35*#1^4 + 21*#1^5
+ 7*#1^6 + #1^7) & ]
```

3.82.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^8(x)+1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^8+1}dx$$
$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\cos^2(x)}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\cos^2(x)+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\cos^2(x)}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\cos^2(x)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \ 3660$$

$$-\frac{1}{4}\int \frac{1}{\left(1-\sqrt[4]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{\left(1+\sqrt[4]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{\left(1-(-1)^{3/4}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{\left(1+(-1)^{3/4}\right)\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \ 216$$

$$-\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

input
```
Int[(1 + Cos[x]^8)^(-1),x]
```

output
```
-1/4*ArcTan[Sqrt[1 - (-1)^(1/4)]*Cot[x]]/Sqrt[1 - (-1)^(1/4)] - ArcTan[Sqr
t[1 + (-1)^(1/4)]*Cot[x]]/(4*Sqrt[1 + (-1)^(1/4)]) - ArcTan[Sqrt[1 - (-1)^
(3/4)]*Cot[x]]/(4*Sqrt[1 - (-1)^(3/4)]) - ArcTan[Sqrt[1 + (-1)^(3/4)]*Cot[
x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

3.82.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.82.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+2\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(8192\_Z^4+(128-128i)\_Z^2+1-i\right)}\_R\ln\left(\mathrm{e}^{2ix}+(1024+1024i)\_R^3+(-128+128i)\_R^2+\right.\right.$

input
```
int(1/(1+cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=Ro
otOf(_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+2))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. 2(89) = 178.

Time = 0.38 (sec) , antiderivative size = 885, normalized size of antiderivative = 6.86

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+cos(x)^8),x, algorithm="fricas")
```

output
```
-1/32*sqrt(2)*sqrt(-sqrt(2*sqrt(2) - 3) - 1)*log(2*(sqrt(2) + 1)*cos(x)^2
+ (2*(sqrt(2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) + 2*(sqrt(2
*sqrt(2) - 3)*(sqrt(2) + 1)*cos(x)*sin(x) + (sqrt(2) + 1)*cos(x)*sin(x))*s
qrt(-sqrt(2*sqrt(2) - 3) - 1) - sqrt(2)) + 1/32*sqrt(2)*sqrt(-sqrt(2*sqrt(
2) - 3) - 1)*log(2*(sqrt(2) + 1)*cos(x)^2 + (2*(sqrt(2) + 2)*cos(x)^2 - sq
rt(2) - 2)*sqrt(2*sqrt(2) - 3) - 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 1)*cos(
x)*sin(x) + (sqrt(2) + 1)*cos(x)*sin(x))*sqrt(-sqrt(2*sqrt(2) - 3) - 1) -
sqrt(2)) - 1/32*sqrt(2)*sqrt(sqrt(2*sqrt(2) - 3) - 1)*log(-2*(sqrt(2) + 1)
*cos(x)^2 + (2*(sqrt(2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) +
 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 1)*cos(x)*sin(x) - (sqrt(2) + 1)*cos(x)
*sin(x))*sqrt(sqrt(2*sqrt(2) - 3) - 1) + sqrt(2)) + 1/32*sqrt(2)*sqrt(sqrt
(2*sqrt(2) - 3) - 1)*log(-2*(sqrt(2) + 1)*cos(x)^2 + (2*(sqrt(2) + 2)*cos(
x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) - 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2)
+ 1)*cos(x)*sin(x) - (sqrt(2) + 1)*cos(x)*sin(x))*sqrt(sqrt(2*sqrt(2) - 3)
 - 1) + sqrt(2)) + 1/32*sqrt(2)*sqrt(-sqrt(-2*sqrt(2) - 3) - 1)*log(2*(sqr
t(2) - 1)*cos(x)^2 + (2*(sqrt(2) - 2)*cos(x)^2 - sqrt(2) + 2)*sqrt(-2*sqrt
(2) - 3) + 2*((sqrt(2) - 1)*sqrt(-2*sqrt(2) - 3)*cos(x)*sin(x) + (sqrt(2)
- 1)*cos(x)*sin(x))*sqrt(-sqrt(-2*sqrt(2) - 3) - 1) - sqrt(2)) - 1/32*sqrt
(2)*sqrt(-sqrt(-2*sqrt(2) - 3) - 1)*log(2*(sqrt(2) - 1)*cos(x)^2 + (2*(sqr
t(2) - 2)*cos(x)^2 - sqrt(2) + 2)*sqrt(-2*sqrt(2) - 3) - 2*((sqrt(2) - ...
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**8),x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \frac{1}{1+\cos^8(x)}\,dx = \int \frac{1}{\cos(x)^8+1}\,dx$$

input
```
integrate(1/(1+cos(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(cos(x)^8 + 1), x)
```

3.82.8 Giac [F]

$$\int \frac{1}{1+\cos^8(x)}\,dx = \int \frac{1}{\cos(x)^8+1}\,dx$$

input
```
integrate(1/(1+cos(x)^8),x, algorithm="giac")
```

output
```
sage0*x
```

3.82.9 Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 1025, normalized size of antiderivative = 7.95

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(x)^8 + 1),x)
```

output
```
atan((tan(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^
(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (2^(1/2)*ta
n(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) -
3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (tan(x)*(2*2^(1/2)
- 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1
/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) + (2^(1/2)*tan(
x)*(2*2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2
^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1))*
((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(x)*(- (2*2^(1/2)
- 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(
1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(- (2*2^(1/2) - 3)^(
1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2
 - (2*2^(1/2) - 3)^(1/2) - 1) + (tan(x)*(2*2^(1/2) - 3)^(1/2)*(- (2*2^(1/2
) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2
^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(2*2^(1/2) - 3)^(1
/2)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) -
 3)^(1/2))/2 + 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1))*(- (2*2^(1/2) - 3)^
(1/2)/128 - 1/128)^(1/2)*2i + atan((tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128
- 1/128)^(1/2)*8i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2
*2^(1/2) - 3)^(1/2) + 1) + (2^(1/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/1...
```

3.82. $\int \frac{1}{1+\cos^8(x)}\,dx$

3.83 $\int \frac{1}{1-\cos^5(x)}\,dx$

3.83.1 Optimal result

Integrand size = 10, antiderivative size = 205

$$\int \frac{1}{1-\cos^5(x)}\,dx = \frac{2\arctan\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}} + \frac{2\arctan\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}} - \frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{-1+(-1)^{4/5}}} + \frac{2\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}} - \frac{\sin(x)}{5(1-\cos(x))}$$

output
```
-1/5*sin(x)/(1-cos(x))+2/5*arctan(((1-(-1)^(3/5))/(1+(-1)^(3/5)))^(1/2)*ta
n(1/2*x))/(1+(-1)^(1/5))^(1/2)+2/5*arctan(((1-(-1)^(1/5))/(1+(-1)^(1/5)))^
(1/2)*tan(1/2*x))/(1-(-1)^(2/5))^(1/2)+2/5*arctanh(((-1-(-1)^(4/5))/(1-(-1
)^(4/5)))^(1/2)*tan(1/2*x))/(-1-(-1)^(3/5))^(1/2)-2/5*arctanh(tan(1/2*x)/(
(-1+(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2))/(-1+(-1)^(4/5))^(1/2)
```

3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.05 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.84

$$\int \frac{1}{1-\cos^5(x)}\,dx$$
$$= -\frac{1}{5}\cot\left(\frac{x}{2}\right)+\frac{1}{10}\text{RootSum}\Big[1+2\#1+8\#1^2+14\#1^3+30\#1^4+14\#1^5+8\#1^6+2\#1^7$$
$$+\#1^8\&,\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)+8\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-4i\log\left(1-2\right.}{}$$

input
```
Integrate[(1 - Cos[x]^5)^(-1),x]
```

output
```
-1/5*Cot[x/2] + RootSum[1 + 2*#1 + 8*#1^2 + 14*#1^3 + 30*#1^4 + 14*#1^5 +
8*#1^6 + 2*#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*C
os[x]*#1 + #1^2] + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - (4*I)*Log[1 - 2*Cos
[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 -
2*Cos[x]*#1 + #1^2]*#1^2 + 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - (40*I)*L
og[1 - 2*Cos[x]*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (
15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1
^5 - (4*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(1 + 8*#1 + 21*#1^2 + 60*#1^
3 + 35*#1^4 + 24*#1^5 + 7*#1^6 + 4*#1^7) & ]/10
```

3.83.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^5(x)}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^5}dx$$
$$\Big\downarrow \; 3692$$
$$\int \left(\frac{1}{5\left(\sqrt[5]{-1}\cos(x)+1\right)}+\frac{1}{5\left(1-(-1)^{2/5}\cos(x)\right)}+\frac{1}{5\left((-1)^{3/5}\cos(x)+1\right)}+\frac{1}{5\left(1-(-1)^{4/5}\cos(x)\right)}+\frac{1}{5(1-\cos$$
$$\Big\downarrow \; 2009$$
$$\frac{2\arctan\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\arctan\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}}-\frac{\sin(x)}{5(1-\cos(x))}$$

input
```
Int[(1 - Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[Sqrt[(1 - (-1)^(1/5))/(1 + (-1)^(1/5))]*Tan[x/2]])/(5*Sqrt[1 - (
-1)^(2/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(3/5))/(1 + (-1)^(3/5))]*Tan[x/2]])
/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(2/5))/(1
 + (-1)^(2/5)))]])/(5*Sqrt[-1 + (-1)^(4/5)]) + (2*ArcTanh[Sqrt[-((1 + (-1)
^(4/5))/(1 - (-1)^(4/5)))]*Tan[x/2]])/(5*Sqrt[-1 - (-1)^(3/5)]) - Sin[x]/(
5*(1 - Cos[x]))
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.83.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+10\_Z^{4}+5\right)}\frac{\left(\_R^{6}+5\_R^{4}+5\_R^{2}+5\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}+5\_R^{3}}\right)}{10}-\frac{1}{5\tan\left(\frac{x}{2}\right)}$
risch	$-\frac{2i}{5\left(\mathrm{e}^{ix}-1\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^{8}+156250\_Z^{6}+6250\_Z^{4}+125\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{ix}+2343750i\_R^{7}-23437\right.\right.$

input
```
int(1/(1-cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/10*sum((_R^6+5*_R^4+5*_R^2+5)/(_R^7+5*_R^3)*ln(tan(1/2*x)-_R),_R=RootOf(
_Z^8+10*_Z^4+5))-1/5/tan(1/2*x)
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. 2(137) = 274.

Time = 0.37 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.65

$$\int \frac{1}{1-\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)^5),x, algorithm="fricas")
```

output
```
1/100*(sqrt(5)*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(sqrt(2*sqrt(5)
*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*sin(x) - 5*
sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*cos(x) - 5*(sqrt(5) - 1)*cos(x) + 20)*si
n(x) - sqrt(5)*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5
)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*sin(x) - 5
*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*cos(x) - 5*(sqrt(5) - 1)*cos(x) + 20)*s
in(x) + sqrt(5)*sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(sqrt(-2*sqrt
(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*sin(x) -
 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*cos(x) + 5*(sqrt(5) - 1)*cos(x) - 20)
*sin(x) - sqrt(5)*sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(-2*s
qrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*sin(x
) - 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*cos(x) + 5*(sqrt(5) - 1)*cos(x) -
20)*sin(x) - sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*log(sqrt(2*
sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) - 5)*si
n(x) - 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5)*cos(x) + 5*(sqrt(5) + 1)*cos(x
) + 20)*sin(x) + sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*log(-sq
rt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) -
5)*sin(x) - 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5)*cos(x) + 5*(sqrt(5) + 1)*
cos(x) + 20)*sin(x) - sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*l
og(sqrt(-2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*s...
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\cos^5(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-cos(x)**5),x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \frac{1}{1-\cos^5(x)}\,dx = \int -\frac{1}{\cos(x)^5 - 1}\,dx$$

input
```
integrate(1/(1-cos(x)^5),x, algorithm="maxima")
```

output
```
1/5*(5*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(2/5*((cos(7*x) + 4*c
os(6*x) + 15*cos(5*x) + 40*cos(4*x) + 15*cos(3*x) + 4*cos(2*x) + cos(x))*c
os(8*x) + (16*cos(6*x) + 44*cos(5*x) + 110*cos(4*x) + 44*cos(3*x) + 16*cos
(2*x) + 4*cos(x) + 1)*cos(7*x) + 2*cos(7*x)^2 + 4*(44*cos(5*x) + 110*cos(4
*x) + 44*cos(3*x) + 16*cos(2*x) + 4*cos(x) + 1)*cos(6*x) + 32*cos(6*x)^2 +
 (1010*cos(4*x) + 420*cos(3*x) + 176*cos(2*x) + 44*cos(x) + 15)*cos(5*x) +
 210*cos(5*x)^2 + 10*(101*cos(3*x) + 44*cos(2*x) + 11*cos(x) + 4)*cos(4*x)
 + 1200*cos(4*x)^2 + (176*cos(2*x) + 44*cos(x) + 15)*cos(3*x) + 210*cos(3*
x)^2 + 4*(4*cos(x) + 1)*cos(2*x) + 32*cos(2*x)^2 + 2*cos(x)^2 + (sin(7*x)
+ 4*sin(6*x) + 15*sin(5*x) + 40*sin(4*x) + 15*sin(3*x) + 4*sin(2*x) + sin(
x))*sin(8*x) + 2*(8*sin(6*x) + 22*sin(5*x) + 55*sin(4*x) + 22*sin(3*x) + 8
*sin(2*x) + 2*sin(x))*sin(7*x) + 2*sin(7*x)^2 + 8*(22*sin(5*x) + 55*sin(4*
x) + 22*sin(3*x) + 8*sin(2*x) + 2*sin(x))*sin(6*x) + 32*sin(6*x)^2 + 2*(50
5*sin(4*x) + 210*sin(3*x) + 88*sin(2*x) + 22*sin(x))*sin(5*x) + 210*sin(5*
x)^2 + 10*(101*sin(3*x) + 44*sin(2*x) + 11*sin(x))*sin(4*x) + 1200*sin(4*x
)^2 + 44*(4*sin(2*x) + sin(x))*sin(3*x) + 210*sin(3*x)^2 + 32*sin(2*x)^2 +
 16*sin(2*x)*sin(x) + 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) + 8*cos(6*x) + 1
4*cos(5*x) + 30*cos(4*x) + 14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(8*
x) + cos(8*x)^2 + 4*(8*cos(6*x) + 14*cos(5*x) + 30*cos(4*x) + 14*cos(3*x)
+ 8*cos(2*x) + 2*cos(x) + 1)*cos(7*x) + 4*cos(7*x)^2 + 16*(14*cos(5*x) ...
```

3.83.8 Giac [F]

$$\int \frac{1}{1-\cos^5(x)}\,dx = \int -\frac{1}{\cos(x)^5 - 1}\,dx$$

input
```
integrate(1/(1-cos(x)^5),x, algorithm="giac")
```

output
```
sage0*x
```

3.83.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.97

$$\int \frac{1}{1-\cos^5(x)}\,dx$$

$$= 2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{-\frac{2\sqrt{5}}{5}-1}+2\sqrt{5}-10\sqrt{-\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$
$$-2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{-\frac{2\sqrt{5}}{5}-1}-2\sqrt{5}-10\sqrt{-\frac{2\sqrt{5}}{5}-1}+5}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$
$$-\frac{\cot\left(\frac{x}{2}\right)}{5}$$
$$+2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}+20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{\frac{2\sqrt{5}}{5}-1}-2\sqrt{5}+10\sqrt{\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$
$$-2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}+20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{\frac{2\sqrt{5}}{5}-1}+2\sqrt{5}+10\sqrt{\frac{2\sqrt{5}}{5}-1}+5}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$

input
```
int(-1/(cos(x)^5 - 1),x)
```

output
```
2*atanh((50*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^
(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(
- (2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) -
5))*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((50*tan(x/2)*(
- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^(1/2)*tan(x/2)*(- (-
 (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(- (2*5^(1/2))/5 -
1)^(1/2) - 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) + 5))*(- (- (2*5^(1/
2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - cot(x/2)/5 + 2*atanh((50*tan(x/2)*(- (
(2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) + 20*5^(1/2)*tan(x/2)*(- ((2*5^(
1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2)
- 2*5^(1/2) + 10*((2*5^(1/2))/5 - 1)^(1/2) - 5))*(- ((2*5^(1/2))/5 - 1)^(1
/2)/50 - 1/50)^(1/2) - 2*atanh((50*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2) + 20*5^(1/2)*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^
(1/2))/(5*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) + 10*((2*5^(1/2))/
5 - 1)^(1/2) + 5))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

3.83. $\int \frac{1}{1-\cos^5(x)}\,dx$

3.84 $\int \frac{1}{1-\cos^6(x)}\,dx$

3.84.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int \frac{1}{1-\cos^6(x)}\,dx=-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{\cot(x)}{3}$$

output
```
-1/3*cot(x)-1/3*arctan(cot(x)*(1+(-1)^(1/3))^(1/2))/(1+(-1)^(1/3))^(1/2)-1
/3*arctan(cot(x)*(1-(-1)^(2/3))^(1/2))/(1-(-1)^(2/3))^(1/2)
```

3.84.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.65

$$\int \frac{1}{1-\cos^6(x)}\,dx$$

$$=\frac{(15+8\cos(2x)+\cos(4x))\sin(x)\left(6\cos(x)+i\sqrt[4]{-3}(3i+\sqrt{3})\arctan\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}(-i+\sqrt{3})\tan(x)\right)\sin(x)\cdots\right.}{144\left(-1+\cos^6(x)\right)}$$

input
```
Integrate[(1 - Cos[x]^6)^(-1),x]
```

output
```
((15 + 8*Cos[2*x] + Cos[4*x])*Sin[x]*(6*Cos[x] + I*(-3)^(1/4)*(3*I + Sqrt[
3])*ArcTan[((-1/3)^(1/4)*(-I + Sqrt[3])*Tan[x])/2]*Sin[x] + (-3)^(1/4)*(-3
*I + Sqrt[3])*ArcTan[((-1)^(3/4)*(I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Sin[x]
))/(144*(-1 + Cos[x]^6))
```

3.84.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^6(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{3}\int \frac{1}{1-\cos^2(x)}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\cos^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \csc^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \csc(x)^2dx$$

$$\Big\downarrow \text{3660}$$

$$-\frac{1}{3}\int \frac{1}{\left(1+\sqrt[3]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{3}\int \frac{1}{\left(1-(-1)^{2/3}\right)\cot^2(x)+1}d\cot(x)+\frac{1}{3}\int \csc(x)^2dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\int \csc(x)^2dx-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(x)}{3}-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{\cot(x)}{3}$$

input
```
Int[(1 - Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[Sqrt[1 + (-1)^(1/3)]*Cot[x]]/Sqrt[1 + (-1)^(1/3)] - ArcTan[Sqr
t[1 - (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 - (-1)^(2/3)]) - Cot[x]/3
```

3.84.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.84.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2i}{3(e^{2ix}-1)}+\left(\sum_{\_R=\operatorname{RootOf}(3888\_Z^4+108\_Z^2+1)}\_R\ln\left(e^{2ix}+1296i\_R^3-216\_R^2-1\right)\right)$
default	$-\frac{\sqrt{3}\left(\frac{\sqrt{2\sqrt{3}-3}\,\ln\left(\tan^2(x)+\tan(x)\sqrt{2\sqrt{3}-3}+\sqrt{3}\right)}{2}+\frac{2\left(-\sqrt{3}-\frac{3}{2}\right)\arctan\left(\frac{2\tan(x)+\sqrt{2\sqrt{3}-3}}{\sqrt{2\sqrt{3}+3}}\right)}{\sqrt{2\sqrt{3}+3}}\right)}{18}-\frac{\sqrt{3}\left(-\frac{\sqrt{2\sqrt{3}-3}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{2\cdots}\right)}{2}\cdots\right)}{\cdots}$

input
```
int(1/(1-cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/(exp(2*I*x)-1)+sum(_R*ln(exp(2*I*x)+1296*I*_R^3-216*_R^2-1),_R=Root
Of(3888*_Z^4+108*_Z^2+1))
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.37

$$\int \frac{1}{1-\cos^6(x)}\,dx = -\frac{\sqrt{6}\sqrt{i\sqrt{3}-3}\log\left(\sqrt{6}\left(i\sqrt{3}-3\right)^{\frac{3}{2}}\cos(x)\sin(x)-6\left(-i\sqrt{3}+2\right)\cos(x)^2-3i\sqrt{3}+3\right)\sin(x)-\sqrt{6}\sqrt{\cdots}}{\cdots}$$

input
```
integrate(1/(1-cos(x)^6),x, algorithm="fricas")
```

output
```
-1/72*(sqrt(6)*sqrt(I*sqrt(3) - 3)*log(sqrt(6)*(I*sqrt(3) - 3)^(3/2)*cos(x
)*sin(x) - 6*(-I*sqrt(3) + 2)*cos(x)^2 - 3*I*sqrt(3) + 3)*sin(x) - sqrt(6)
*sqrt(I*sqrt(3) - 3)*log(sqrt(6)*sqrt(I*sqrt(3) - 3)*(-I*sqrt(3) + 3)*cos(
x)*sin(x) - 6*(-I*sqrt(3) + 2)*cos(x)^2 - 3*I*sqrt(3) + 3)*sin(x) + sqrt(6
)*sqrt(-I*sqrt(3) - 3)*log(sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 3)*co
s(x)*sin(x) - 6*(-I*sqrt(3) - 2)*cos(x)^2 - 3*I*sqrt(3) - 3)*sin(x) - sqrt
(6)*sqrt(-I*sqrt(3) - 3)*log(sqrt(6)*(-I*sqrt(3) - 3)^(3/2)*cos(x)*sin(x)
- 6*(-I*sqrt(3) - 2)*cos(x)^2 - 3*I*sqrt(3) - 3)*sin(x) + 24*cos(x))/sin(x
)
```

3.84.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(66) = 132$.

Time = 8.91 (sec) , antiderivative size = 728, normalized size of antiderivative = 10.25

$$\int \frac{1}{1-\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)**6),x)
```

output
```
sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*floor((x/2 - pi
/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*f
loor((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/
2) + 1) + pi*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3
**(1/4)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(
atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi/2)/pi))/36 + sq
rt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi
/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 + 1) + pi
*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(
x/2)/3 + 1) + pi*floor((x/2 - pi/2)/pi))/12 - sqrt(2)*3**(1/4)*log(4*tan(x
/2)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/24 + sqrt(2)*3**(3/4)*lo
g(4*tan(x/2)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 - sqrt(2)*3*
*(3/4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 + s
qrt(2)*3**(1/4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3
))/24 - sqrt(2)*3**(3/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2)
 + 12*sqrt(3))/72 + sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3
/4)*tan(x/2) + 12*sqrt(3))/24 - sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 + 12*s
qrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/24 + sqrt(2)*3**(3/4)*log(36*tan(x/
2)**2 + 12*sqrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/72 + tan(x/2)/6 - 1/(6*
tan(x/2))
```

3.84.7 Maxima [F]

$$\int \frac{1}{1-\cos^6(x)}\,dx = \int -\frac{1}{\cos{(x)}^6-1}\,dx$$

input
```
integrate(1/(1-cos(x)^6),x, algorithm="maxima")
```

output
```
1/3*(3*(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(1/3*((cos(3*x)
 + 4*cos(2*x) + cos(x))*cos(4*x) + (14*cos(2*x) + 4*cos(x) + 1)*cos(3*x) +
 2*cos(3*x)^2 + 2*(7*cos(x) + 2)*cos(2*x) + 24*cos(2*x)^2 + 2*cos(x)^2 + (
sin(3*x) + 4*sin(2*x) + sin(x))*sin(4*x) + 2*(7*sin(2*x) + 2*sin(x))*sin(3
*x) + 2*sin(3*x)^2 + 24*sin(2*x)^2 + 14*sin(2*x)*sin(x) + 2*sin(x)^2 + cos
(x))/(2*(2*cos(3*x) + 6*cos(2*x) + 2*cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 4
*(6*cos(2*x) + 2*cos(x) + 1)*cos(3*x) + 4*cos(3*x)^2 + 12*(2*cos(x) + 1)*c
os(2*x) + 36*cos(2*x)^2 + 4*cos(x)^2 + 4*(sin(3*x) + 3*sin(2*x) + sin(x))*
sin(4*x) + sin(4*x)^2 + 8*(3*sin(2*x) + sin(x))*sin(3*x) + 4*sin(3*x)^2 +
36*sin(2*x)^2 + 24*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1), x) - 3*(c
os(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(-1/3*((cos(3*x) - 4*cos
(2*x) + cos(x))*cos(4*x) + (14*cos(2*x) - 4*cos(x) + 1)*cos(3*x) - 2*cos(3
*x)^2 + 2*(7*cos(x) - 2)*cos(2*x) - 24*cos(2*x)^2 - 2*cos(x)^2 + (sin(3*x)
 - 4*sin(2*x) + sin(x))*sin(4*x) + 2*(7*sin(2*x) - 2*sin(x))*sin(3*x) - 2*
sin(3*x)^2 - 24*sin(2*x)^2 + 14*sin(2*x)*sin(x) - 2*sin(x)^2 + cos(x))/(2*
(2*cos(3*x) - 6*cos(2*x) + 2*cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 4*(6*cos(
2*x) - 2*cos(x) + 1)*cos(3*x) - 4*cos(3*x)^2 + 12*(2*cos(x) - 1)*cos(2*x)
- 36*cos(2*x)^2 - 4*cos(x)^2 + 4*(sin(3*x) - 3*sin(2*x) + sin(x))*sin(4*x)
 - sin(4*x)^2 + 8*(3*sin(2*x) - sin(x))*sin(3*x) - 4*sin(3*x)^2 - 36*sin(2
*x)^2 + 24*sin(2*x)*sin(x) - 4*sin(x)^2 + 4*cos(x) - 1), x) - 2*sin(2*x...
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(49) = 98$.

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.80

$$\int \frac{1}{1-\cos^6(x)}\,dx$$
$$= \frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor - \arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan(x)\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}\right)\right)\sqrt{6\sqrt{3}+9}$$
$$+ \frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor + \arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)-4\tan(x)\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}\right)\right)\sqrt{6\sqrt{3}+9}$$
$$- \frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(\frac{1}{2}\left(\sqrt{6}3^{\frac{1}{4}}-3^{\frac{1}{4}}\sqrt{2}\right)\tan(x)+\tan(x)^2+\sqrt{3}\right)$$
$$+ \frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(-\frac{1}{2}\left(\sqrt{6}3^{\frac{1}{4}}-3^{\frac{1}{4}}\sqrt{2}\right)\tan(x)+\tan(x)^2+\sqrt{3}\right) - \frac{1}{3\tan(x)}$$

input
```
integrate(1/(1-cos(x)^6),x, algorithm="giac")
```

output
```
1/18*(pi*floor(x/pi + 1/2) - arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(
2)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi*floor
(x/pi + 1/2) + arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(2)) - 4*tan(x)
)/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) - 1/36*sqrt(6*sqrt(3) - 9)*log
(1/2*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x) + tan(x)^2 + sqrt(3)) + 1/
36*sqrt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x)
 + tan(x)^2 + sqrt(3)) - 1/3/tan(x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{1}{1-\cos^6(x)}\,dx = -\frac{1}{3\tan(x)} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\sqrt{6}\tan(x)\left(\frac{1}{27}-\frac{1}{27}\mathrm{i}\right)}{-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\left(3^{1/4}\left(1+1\mathrm{i}\right)+3^{3/4}\left(-1+1\mathrm{i}\right)\right)1\mathrm{i}}{36} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\sqrt{6}\tan(x)\left(\frac{1}{27}+\frac{1}{27}\mathrm{i}\right)}{\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\left(3^{1/4}\left(1-\mathrm{i}\right)+3^{3/4}\left(-1-\mathrm{i}\right)\right)1\mathrm{i}}{36}$$

input
```
int(-1/(cos(x)^6 - 1),x)
```

output
```
(6^(1/2)*atan((3^(1/4)*6^(1/2)*tan(x)*(1/27 - 1i/27))/((3^(1/2)*1i)/9 - 1/
9))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 - 1/(3*tan(x)) + (6^(1/2)
*atan((3^(1/4)*6^(1/2)*tan(x)*(1/27 + 1i/27))/((3^(1/2)*1i)/9 + 1/9))*(3^(
1/4)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

3.85 $\int \frac{1}{1-\cos^8(x)}\,dx$

3.85.1 Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{1}{1-\cos^8(x)}\,dx=\frac{x}{4\sqrt{2}}-\frac{\arctan\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\cot(x)}{4}$$

output
```
-1/4*cot(x)-1/4*arctan(cot(x)*(1-I)^(1/2))/(1-I)^(1/2)-1/4*arctan(cot(x)*(
1+I)^(1/2))/(1+I)^(1/2)+1/8*x*2^(1/2)-1/8*arctan(cos(x)*sin(x)/(1+cos(x)^2
+2^(1/2)))*2^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \frac{1}{1-\cos^8(x)}\,dx=\frac{1}{8}\left(\frac{2\arctan\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{2\arctan\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{\sqrt{1+i}}+\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)-2\cot(x)\right)$$

input
```
Integrate[(1 - Cos[x]^8)^(-1),x]
```

output
```
((2*ArcTan[Tan[x]/Sqrt[1 - I]])/Sqrt[1 - I] + (2*ArcTan[Tan[x]/Sqrt[1 + I]
])/Sqrt[1 + I] + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/8
```

3.85.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^8(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\cos^2(x)}dx+\frac{1}{4}\int \frac{1}{1-i\cos^2(x)}dx+\frac{1}{4}\int \frac{1}{i\cos^2(x)+1}dx+\frac{1}{4}\int \frac{1}{\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{1-i\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \csc^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \csc(x)^2dx$$

$$\Big\downarrow \text{3660}$$

$$-\frac{1}{4}\int \frac{1}{(1-i)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{(1+i)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{2\cot^2(x)+1}d\cot(x)+\frac{1}{4}\int \csc(x)^2dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\int \csc(x)^2dx-\frac{\arctan\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}}-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(x)}{4}-\frac{\arctan\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}}-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\arctan\left(\sqrt{1-i}\cot(x)\right)}{4\sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i}\cot(x)\right)}{4\sqrt{1+i}}-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}}-\frac{\cot(x)}{4}$$

input
```
Int[(1 - Cos[x]^8)^(-1),x]
```

output
```
-1/4*ArcTan[Sqrt[1 - I]*Cot[x]]/Sqrt[1 - I] - ArcTan[Sqrt[1 + I]*Cot[x]]/(
4*Sqrt[1 + I]) - ArcTan[Sqrt[2]*Cot[x]]/(4*Sqrt[2]) - Cot[x]/4
```

3.85.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.85.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(65) = 130.

Time = 2.68 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{i}{2(e^{2ix}-1)}+\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}-i\sqrt{-2-2i}+1+2i+\sqrt{-2-2i}\right)}{16}-\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}+i\sqrt{-2-2i}+1+2i-\sqrt{-2-2i}\right)}{16}+\sqrt{-2+2i}\,\ln$
default	$-\frac{1}{4\tan(x)}-\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{2\left(-1-\sqrt{2}\right)\arctan\left(\frac{2\tan(x)-\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right)}{\sqrt{2\sqrt{2}+2}}\right)}{16}-\sqrt{2}\left(\sqrt{-2+2\sqrt{2}}\,\ln\right.$

input
```
int(1/(1-cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/(exp(2*I*x)-1)+1/16*(-2-2*I)^(1/2)*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+1
+2*I+(-2-2*I)^(1/2))-1/16*(-2-2*I)^(1/2)*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)+1+
2*I-(-2-2*I)^(1/2))+1/16*(-2+2*I)^(1/2)*ln(exp(2*I*x)-I*(-2+2*I)^(1/2)-(-2
+2*I)^(1/2)+1-2*I)-1/16*(-2+2*I)^(1/2)*ln(exp(2*I*x)+I*(-2+2*I)^(1/2)+(-2+
2*I)^(1/2)+1-2*I)+1/16*I*2^(1/2)*ln(exp(2*I*x)+2*2^(1/2)+3)-1/16*I*2^(1/2)
*ln(exp(2*I*x)-2*2^(1/2)+3)
```

3.85.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(57) = 114$.

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.87

$$\int \frac{1}{1-\cos^8(x)}\,dx$$
$$= \frac{\sqrt{2}\sqrt{i-1}\log\left(-(i-1)\sqrt{2}\sqrt{i-1}\cos(x)\sin(x)+(2i-1)\cos(x)^2-i\right)\sin(x)-\sqrt{2}\sqrt{i-1}\log\left((i-1)\right.}{}$$

input
```
integrate(1/(1-cos(x)^8),x, algorithm="fricas")
```

output
```
1/32*(sqrt(2)*sqrt(I - 1)*log(-(I - 1)*sqrt(2)*sqrt(I - 1)*cos(x)*sin(x) +
 (2*I - 1)*cos(x)^2 - I)*sin(x) - sqrt(2)*sqrt(I - 1)*log((I - 1)*sqrt(2)*
sqrt(I - 1)*cos(x)*sin(x) + (2*I - 1)*cos(x)^2 - I)*sin(x) - sqrt(2)*sqrt(
-I - 1)*log((I + 1)*sqrt(2)*sqrt(-I - 1)*cos(x)*sin(x) + (2*I + 1)*cos(x)^
2 - I)*sin(x) + sqrt(2)*sqrt(-I - 1)*log(-(I + 1)*sqrt(2)*sqrt(-I - 1)*cos
(x)*sin(x) + (2*I + 1)*cos(x)^2 - I)*sin(x) - 2*sqrt(2)*arctan(1/4*(3*sqrt
(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))*sin(x) - 8*cos(x))/sin(x)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\cos^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-cos(x)**8),x)
```

output
```
Timed out
```

3.85.7 Maxima [F]

$$\int \frac{1}{1-\cos^8(x)}\,dx = \int -\frac{1}{\cos(x)^8-1}\,dx$$

input
```
integrate(1/(1-cos(x)^8),x, algorithm="maxima")
```

output
```
1/8*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*arctan2(4*sqrt(2)*sin(2*x)
/(2*(2*sqrt(2) + 3)*cos(2*x) + cos(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17),
 (cos(2*x)^2 + sin(2*x)^2 + 6*cos(2*x) + 1)/(2*(2*sqrt(2) + 3)*cos(2*x) +
cos(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17)) + 64*(sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))*integrate(((4*cos(2*x) + 1
)*cos(4*x) + cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) + 22*cos(4*x)^2 + sin
(8*x)*sin(4*x) + 4*sin(6*x)*sin(4*x) + 22*sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
)/(2*(4*cos(6*x) + 22*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8
*(22*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 44*(4*cos(2*x)
+ 1)*cos(4*x) + 484*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 11*sin(4*
x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(11*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 484*sin(4*x)^2 + 176*sin(4*x)*sin(2*x) + 16*sin(2*
x)^2 + 8*cos(2*x) + 1), x) - 4*sqrt(2)*sin(2*x))/(sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(57) = 114$.

Time = 0.54 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.49

$$\begin{aligned}
&\int \frac{1}{1-\cos^8(x)}\,dx \\
&= \frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right) \\
&\quad+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1} \\
&\quad+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1} \\
&\quad-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right) \\
&\quad+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)-\frac{1}{4\tan(x)}
\end{aligned}$$

input

```
integrate(1/(1-cos(x)^8),x, algorithm="giac")
```

output

```
1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4
)*(2^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2)
 + 1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqr
t(2) + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) - 1/16*sqrt(sq
rt(2) - 1)*log(tan(x)^2 + 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1
/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 - 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + s
qrt(2)) - 1/4/tan(x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.71

$$\int \frac{1}{1-\cos^8(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8} - \mathrm{atan}\left(\frac{\sqrt{2}\tan(x)\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)} + \frac{\sqrt{2}\tan(x)\,\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i} - \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right) + \mathrm{atan}\left(\frac{\sqrt{2}\tan(x)\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}+\frac{1}{16}\right)} - \frac{\sqrt{2}\tan(x)\,\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}+\frac{1}{16}\right)}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i} + \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right) - \frac{1}{4\tan(x)}$$

input
```
int(-1/(cos(x)^8 - 1),x)
```

output
```
atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256
 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) + 1/16)) - (2^(1/2)*tan(x)*(
2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/
2)/256 - 1/256)^(1/2) + 1/16)))*((- 2^(1/2)/256 - 1/256)^(1/2)*2i + (2^(1/
2)/256 - 1/256)^(1/2)*2i) - atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(
1/2)*1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2)
- 1/16)) + (2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)
/256 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) - 1/16)))*((- 2^(1/2)/25
6 - 1/256)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) - 1/(4*tan(x)) + (2^
(1/2)*atan((2^(1/2)*tan(x))/2))/8
```

3.86 $\int \frac{\tan(x)}{1+\cos^2(x)} dx$

3.86.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\tan(x)}{1+\cos^2(x)} dx = -\log(\cos(x)) + \frac{1}{2}\log\left(1+\cos^2(x)\right)$$

output
```
-ln(cos(x))+1/2*ln(1+cos(x)^2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{1+\cos^2(x)} dx = -\log(\cos(x)) + \frac{1}{2}\log\left(1+\cos^2(x)\right)$$

input
```
Integrate[Tan[x]/(1 + Cos[x]^2),x]
```

output
```
-Log[Cos[x]] + Log[1 + Cos[x]^2]/2
```

3.86.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 25, 3673, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3673}$$

$$-\frac{1}{2}\int \frac{\sec^2(x)}{\cos^2(x)+1}d\cos^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{\cos^2(x)+1}d\cos^2(x)-\int \sec^2(x)d\cos^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\int \frac{1}{\cos^2(x)+1}d\cos^2(x)-\log\left(\cos^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(\cos^2(x)+1\right)-\log\left(\cos^2(x)\right)\right)$$

input
```
Int[Tan[x]/(1 + Cos[x]^2),x]
```

output
```
(-Log[Cos[x]^2] + Log[1 + Cos[x]^2])/2
```

3.86.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.86.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln\left(\cos\left(x\right)\right)+\frac{\ln\left(1+\cos^{2}\left(x\right)\right)}{2}$	16
risch	$-\ln\left({\mathrm e}^{2 i x}+1\right)+\frac{\ln\left({\mathrm e}^{4 i x}+6 \,{\mathrm e}^{2 i x}+1\right)}{2}$	29

input
```
int(tan(x)/(1+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-ln(cos(x))+1/2*ln(1+cos(x)^2)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\tan(x)}{1+\cos^2(x)}\,dx = \frac{1}{2}\log\left(\frac{1}{2}\cos(x)^2+\frac{1}{2}\right)-\log(-\cos(x))$$

input
```
integrate(tan(x)/(1+cos(x)^2),x, algorithm="fricas")
```

output
```
1/2*log(1/2*cos(x)^2 + 1/2) - log(-cos(x))
```

3.86.6 Sympy [F]

$$\int \frac{\tan(x)}{1+\cos^2(x)}\,dx = \int \frac{\tan(x)}{\cos^2(x)+1}\,dx$$

input
```
integrate(tan(x)/(1+cos(x)**2),x)
```

output
```
Integral(tan(x)/(cos(x)**2 + 1), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\tan(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\log\left(\sin(x)^2-1\right)+\frac{1}{2}\log\left(\sin(x)^2-2\right)$$

input
```
integrate(tan(x)/(1+cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 2)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\tan(x)}{1+\cos^2(x)}\,dx = \frac{1}{2}\,\log\left(\cos\left(x\right)^2+1\right) - \log\left(\left|\cos\left(x\right)\right|\right)$$

input
```
integrate(tan(x)/(1+cos(x)^2),x, algorithm="giac")
```

output
```
1/2*log(cos(x)^2 + 1) - log(abs(cos(x)))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\tan(x)}{1+\cos^2(x)}\,dx = \frac{\ln\left(\tan(x)^2+2\right)}{2}$$

input
```
int(tan(x)/(cos(x)^2 + 1),x)
```

output
```
log(tan(x)^2 + 2)/2
```

3.87 $\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx$

3.87.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx=\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\cos^2(x)}$$

output
```
arctanh((a+b*cos(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*cos(x)^2)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx=\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\cos^2(x)}$$

input
```
Integrate[Sqrt[a + b*Cos[x]^2]*Tan[x],x]
```

output
```
Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]] - Sqrt[a + b*Cos[x]^2]
```

3.87.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3673, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^2}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3673$$

$$-\frac{1}{2}\int \sqrt{b\cos^2(x)+a}\sec^2(x)d\cos^2(x)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(-a\int \frac{\sec^2(x)}{\sqrt{b\cos^2(x)+a}}d\cos^2(x)-2\sqrt{a+b\cos^2(x)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{\cos^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cos^2(x)+a}}{b}-2\sqrt{a+b\cos^2(x)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-2\sqrt{a+b\cos^2(x)}\right)$$

input
```
Int[Sqrt[a + b*Cos[x]^2]*Tan[x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]] - 2*Sqrt[a + b*Cos[x]^2])
/2
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.87.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

method	result	size
default	$-\sqrt{a+b\left(\cos^{2}\left(x\right)\right)}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^{2}\left(x\right)\right)}}{\cos\left(x\right)}\right)$	43

input
```
int((a+b*cos(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*cos(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^2)^(1/2))/cos(
x))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int\sqrt{a+b\cos^{2}(x)}\tan(x)\,dx=\left[\frac{1}{2}\sqrt{a}\log\left(\frac{b\cos\left(x\right)^{2}+2\sqrt{b\cos\left(x\right)^{2}+a}\sqrt{a}+2\,a}{\cos\left(x\right)^{2}}\right)-\sqrt{b\cos\left(x\right)^{2}+a},-\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{2}+a}\sqrt{-a}}{a}\right)-\sqrt{b\cos\left(x\right)^{2}+a}\right]$$

input
```
integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((b*cos(x)^2 + 2*sqrt(b*cos(x)^2 + a)*sqrt(a) + 2*a)/cos(x
)^2) - sqrt(b*cos(x)^2 + a), -sqrt(-a)*arctan(sqrt(b*cos(x)^2 + a)*sqrt(-a
)/a) - sqrt(b*cos(x)^2 + a)]
```

3.87.6 Sympy [F]

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx = \int \sqrt{a+b\cos^2{(x)}}\tan{(x)}\,dx$$

input
```
integrate((a+b*cos(x)**2)**(1/2)*tan(x),x)
```

output
```
Integral(sqrt(a + b*cos(x)**2)*tan(x), x)
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.38

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx = \frac{1}{2}\sqrt{a}\log\left(b-\frac{\sqrt{-b\sin{(x)}^2+a+b}\sqrt{a}}{\sin{(x)}-1}-\frac{a}{\sin{(x)}-1}\right) + \frac{1}{2}\sqrt{a}\log\left(-b+\frac{\sqrt{-b\sin{(x)}^2+a+b}\sqrt{a}}{\sin{(x)}+1}+\frac{a}{\sin{(x)}+1}\right) - \sqrt{-b\sin{(x)}^2+a+b}$$

input
```
integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*log(b - sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) - 1) - a/(si
n(x) - 1)) + 1/2*sqrt(a)*log(-b + sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x
) + 1) + a/(sin(x) + 1)) - sqrt(-b*sin(x)^2 + a + b)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx = -\frac{a\arctan\left(\frac{\sqrt{b\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \sqrt{b\cos(x)^2+a}$$

input
```
integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

output
```
-a*arctan(sqrt(b*cos(x)^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*cos(x)^2 + a)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx = \int \tan(x)\,\sqrt{b\cos(x)^2+a}\,dx$$

input
```
int(tan(x)*(a + b*cos(x)^2)^(1/2),x)
```

output
```
int(tan(x)*(a + b*cos(x)^2)^(1/2), x)
```

3.88 $\int \sqrt{1-\cos^2(x)}\tan(x)\,dx$

3.88.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = \operatorname{arctanh}\left(\sqrt{\sin^2(x)}\right)-\sqrt{\sin^2(x)}$$

output
```
arctanh((sin(x)^2)^(1/2))-(sin(x)^2)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = (-1+\operatorname{arctanh}(\sin(x))\csc(x))\sqrt{\sin^2(x)}$$

input
```
Integrate[Sqrt[1 - Cos[x]^2]*Tan[x],x]
```

output
```
(-1 + ArcTanh[Sin[x]]*Csc[x])*Sqrt[Sin[x]^2]
```

3.88.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 25, 3655, 25, 3042, 3684, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\sqrt{\sin^2(x)}\tan(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int \sqrt{\sin^2(x)}\tan(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)^2}\tan(x)dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{1}{2}\int \frac{\sqrt{\sin^2(x)}}{1-\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin^2(x)-2\sqrt{\sin^2(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\int \frac{1}{1-\sin^4(x)}d\sqrt{\sin^2(x)}-2\sqrt{\sin^2(x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(\sqrt{\sin^2(x)}\right)-2\sqrt{\sin^2(x)}\right)$$

input
```
Int[Sqrt[1 - Cos[x]^2]*Tan[x],x]
```

output
```
(2*ArcTanh[Sqrt[Sin[x]^2]] - 2*Sqrt[Sin[x]^2])/2
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.88.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result
default	$-\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}+\operatorname{arctanh}\left(\frac{2}{\sqrt{2-2\cos(2x)}}\right)$
risch	$-\frac{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}\, e^{2ix}}{2(e^{2ix}-1)}+\frac{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}{2\, e^{2ix}-2}-\frac{i\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}\, e^{ix} \ln(e^{ix}-i)}{e^{2ix}-1}+\frac{i\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}\, e^{ix} \ln(e^{ix}+i)}{e^{2ix}-1}$

input
```
int((1-cos(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
-(sin(x)^2)^(1/2)+arctanh(1/(sin(x)^2)^(1/2))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = \frac{1}{2}\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

input
```
integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

3.88.6 Sympy [F]

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = \int \sqrt{-\left(\cos\left(x\right)-1\right)\left(\cos\left(x\right)+1\right)}\tan\left(x\right)dx$$

input
```
integrate((1-cos(x)**2)**(1/2)*tan(x),x)
```

output
```
Integral(sqrt(-(cos(x) - 1)*(cos(x) + 1))*tan(x), x)
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = \frac{1}{2}\left(-1\right)^{2\sin\left(x\right)}\log\left(-\frac{\sin\left(x\right)}{\sin\left(x\right)+1}\right) + \frac{1}{2}\left(-1\right)^{2\sin\left(x\right)}\log\left(-\frac{\sin\left(x\right)}{\sin\left(x\right)-1}\right) - \sqrt{\sin\left(x\right)^2}$$

input
```
integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
1/2*(-1)^(2*sin(x))*log(-sin(x)/(sin(x) + 1)) + 1/2*(-1)^(2*sin(x))*log(-s
in(x)/(sin(x) - 1)) - sqrt(sin(x)^2)
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = -\sqrt{-\cos\left(x\right)^2+1} + \frac{1}{2}\log\left(\sqrt{-\cos\left(x\right)^2+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\cos\left(x\right)^2+1}+1\right)$$

input
```
integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

output
```
-sqrt(-cos(x)^2 + 1) + 1/2*log(sqrt(-cos(x)^2 + 1) + 1) - 1/2*log(-sqrt(-c
os(x)^2 + 1) + 1)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\cos^2(x)}\tan(x)\,dx = \int \tan(x)\,\sqrt{1-{\cos\left(x\right)}^2}\,dx$$

input
```
int(tan(x)*(1 - cos(x)^2)^(1/2),x)
```

output
```
int(tan(x)*(1 - cos(x)^2)^(1/2), x)
```

3.89 $\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx$

3.89.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
arctanh((a+b*cos(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Tan[x]/Sqrt[a + b*Cos[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]]/Sqrt[a]
```

3.89.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3673, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}\tan\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \; 3673$$
$$-\frac{1}{2}\int \frac{\sec^2(x)}{\sqrt{b\cos^2(x)+a}}d\cos^2(x)$$
$$\Big\downarrow \; 73$$
$$-\frac{\int \frac{1}{\frac{\cos^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cos^2(x)+a}}{b}$$
$$\Big\downarrow \; 221$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Tan[x]/Sqrt[a + b*Cos[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]]/Sqrt[a]
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.89.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^2(x)\right)}}{\cos(x)}\right)}{\sqrt{a}}$	30

input
```
int(tan(x)/(a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^2)^(1/2))/cos(x))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx = \left[\frac{\log\left(\frac{b\cos(x)^2+2\sqrt{b\cos(x)^2+a}\sqrt{a}+2a}{\cos(x)^2}\right)}{2\sqrt{a}}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^2+a}\sqrt{-a}}{a}\right)}{a}\right]$$

input
```
integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b*cos(x)^2 + 2*sqrt(b*cos(x)^2 + a)*sqrt(a) + 2*a)/cos(x)^2)/sqr
t(a), -sqrt(-a)*arctan(sqrt(b*cos(x)^2 + a)*sqrt(-a)/a)/a]
```

3.89.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx$$

input
```
integrate(tan(x)/(a+b*cos(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a + b*cos(x)**2), x)
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(19) = 38.

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx = \frac{\log\left(b-\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)-1}-\frac{a}{\sin(x)-1}\right)}{2\sqrt{a}} + \frac{\log\left(-b+\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)+1}+\frac{a}{\sin(x)+1}\right)}{2\sqrt{a}}$$

input
```
integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(b - sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) - 1) - a/(sin(x) - 1
))/sqrt(a) + 1/2*log(-b + sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) + 1) +
 a/(sin(x) + 1))/sqrt(a)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-arctan(sqrt(b*cos(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{b\cos(x)^2+a}}\,dx$$

input
```
int(tan(x)/(a + b*cos(x)^2)^(1/2),x)
```

output
```
int(tan(x)/(a + b*cos(x)^2)^(1/2), x)
```

3.90 $\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

3.90.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \operatorname{arctanh}\left(\sqrt{1+\cos^2(x)}\right)$$

output
```
arctanh((1+cos(x)^2)^(1/2))
```

3.90.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \operatorname{arctanh}\left(\sqrt{1+\cos^2(x)}\right)$$

input
```
Integrate[Tan[x]/Sqrt[1 + Cos[x]^2],x]
```

output
```
ArcTanh[Sqrt[1 + Cos[x]^2]]
```

3.90.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 25, 3673, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{\cos^2(x)+1}}dx$$
$$\Big\downarrow \ 3042$$
$$\int -\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}\tan\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 25$$
$$-\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}\tan\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 3673$$
$$-\frac{1}{2}\int \frac{\sec^2(x)}{\sqrt{\cos^2(x)+1}}d\cos^2(x)$$
$$\Big\downarrow \ 73$$
$$-\int \frac{1}{\cos^4(x)-1}d\sqrt{\cos^2(x)+1}$$
$$\Big\downarrow \ 220$$
$$\text{arctanh}\left(\sqrt{\cos^2(x)+1}\right)$$

input
```
Int[Tan[x]/Sqrt[1 + Cos[x]^2],x]
```

output
```
ArcTanh[Sqrt[1 + Cos[x]^2]]
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.90.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\cos^{2}(x)}}\right)$	10

input
```
int(tan(x)/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(1/(1+cos(x)^2)^(1/2))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \log\left(\frac{\sqrt{\cos(x)^2+1}+1}{\cos(x)}\right)$$

input
```
integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log((sqrt(cos(x)^2 + 1) + 1)/cos(x))
```

3.90.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

input
```
integrate(tan(x)/(1+cos(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(cos(x)**2 + 1), x)
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(9) = 18.

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.45

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{1}{2}\log\left(\frac{\sqrt{-\sin(x)^2+2}}{\sin(x)+1}+\frac{1}{\sin(x)+1}-1\right) + \frac{1}{2}\log\left(-\frac{\sqrt{-\sin(x)^2+2}}{\sin(x)-1}-\frac{1}{\sin(x)-1}+1\right)$$

input
```
integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(sqrt(-sin(x)^2 + 2)/(sin(x) + 1) + 1/(sin(x) + 1) - 1) + 1/2*log(-
sqrt(-sin(x)^2 + 2)/(sin(x) - 1) - 1/(sin(x) - 1) + 1)
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(9) = 18$.

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{1}{2}\log\left(\sqrt{\cos(x)^2+1}+1\right) - \frac{1}{2}\log\left(\sqrt{\cos(x)^2+1}-1\right)$$

input
```
integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(sqrt(cos(x)^2 + 1) + 1) - 1/2*log(sqrt(cos(x)^2 + 1) - 1)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{\cos(x)^2+1}}\,dx$$

input
```
int(tan(x)/(cos(x)^2 + 1)^(1/2),x)
```

output
```
int(tan(x)/(cos(x)^2 + 1)^(1/2), x)
```

3.91 $\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx$

3.91.1 Optimal result

Integrand size = 15, antiderivative size = 9

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \operatorname{arctanh}\left(\sqrt{\sin^2(x)}\right)$$

output
```
arctanh((sin(x)^2)^(1/2))
```

3.91.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\sin(x)}{\sqrt{\sin^2(x)}}$$

input
```
Integrate[Tan[x]/Sqrt[1 - Cos[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Sin[x])/Sqrt[Sin[x]^2]
```

3.91.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 25, 3655, 25, 3042, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3655}$$

$$-\int -\frac{\tan(x)}{\sqrt{\sin^2(x)}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\tan(x)}{\sqrt{\sin^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(x)}{\sqrt{\sin(x)^2}}dx$$

$$\Big\downarrow \text{ 3684}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin^2(x)$$

$$\Big\downarrow \text{ 73}$$

$$\int \frac{1}{1-\sin^4(x)}d\sqrt{\sin^2(x)}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\sqrt{\sin^2(x)}\right)$$

input
```
Int[Tan[x]/Sqrt[1 - Cos[x]^2],x]
```

output
```
ArcTanh[Sqrt[Sin[x]^2]]
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.91.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$\operatorname{arctanh}\left(\frac{2}{\sqrt{2-2\cos(2x)}}\right)$	8
risch	$\frac{2\ln(e^{ix}+i)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}-\frac{2\ln(e^{ix}-i)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}$	64

input
```
int(tan(x)/(1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(1/(sin(x)^2)^(1/2))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)$$

input
```
integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.91.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{-(\cos(x)-1)(\cos(x)+1)}}\,dx$$

input
```
integrate(tan(x)/(1-cos(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(-(cos(x) - 1)*(cos(x) + 1)), x)
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(7) = 14.

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 4.33

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \frac{1}{2}(-1)^{2\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)+1}\right) + \frac{1}{2}(-1)^{2\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)-1}\right)$$

input
```
integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(-1)^(2*sin(x))*log(-sin(x)/(sin(x) + 1)) + 1/2*(-1)^(2*sin(x))*log(-s
in(x)/(sin(x) - 1))
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(7) = 14.

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.67

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \frac{1}{2}\log\left(\sqrt{-\cos(x)^2+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\cos(x)^2+1}+1\right)$$

input
```
integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(sqrt(-cos(x)^2 + 1) + 1) - 1/2*log(-sqrt(-cos(x)^2 + 1) + 1)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{1-\cos(x)^2}}\,dx$$

input
```
int(tan(x)/(1 - cos(x)^2)^(1/2),x)
```

output
```
int(tan(x)/(1 - cos(x)^2)^(1/2), x)
```

3.92 $\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx$

3.92.1 Optimal result

Integrand size = 15, antiderivative size = 153

$$\begin{aligned}\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = &-\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cos(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} \\ &+\frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}} \\ &-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\cos^2(x)\right)}{6a^{5/3}} \\ &-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}\end{aligned}$$

output
```
ln(cos(x))/a+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*cos(x))/a^(5/3)-1/6*b^(2/3)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*cos(x)+b^(2/3)*cos(x)^2)/a^(5/3)-1/3*ln(a+b*cos(x
)^3)/a+1/2*sec(x)^2/a-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*cos(x))/a^
(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.42

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx$$
$$= \frac{6\left(\log(\cos(x))+\log\left(\sec^2\left(\frac{x}{2}\right)\right)\right)-2\text{RootSum}\left[a+b+3a\#1-3b\#1+3a\#1^2+3b\#1^2+a\#1^3-b\#1^3\&\right.}{}$$

input
```
Integrate[Tan[x]^3/(a + b*Cos[x]^3),x]
```

output
```
(6*(Log[Cos[x]] + Log[Sec[x/2]^2]) - 2*RootSum[a + b + 3*a*#1 - 3*b*#1 + 3
*a*#1^2 + 3*b*#1^2 + a*#1^3 - b*#1^3 & , (a*Log[-#1 + Tan[x/2]^2] + b*Log[
-#1 + Tan[x/2]^2] + 2*a*Log[-#1 + Tan[x/2]^2]*#1 + 4*b*Log[-#1 + Tan[x/2]^
2]*#1 + a*Log[-#1 + Tan[x/2]^2]*#1^2 - b*Log[-#1 + Tan[x/2]^2]*#1^2)/(a -
b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] + 3*Sec[x]^2)/(6*a)
```

3.92.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 3709, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)^3\left(a+b\sin\left(x+\frac{\pi}{2}\right)^3\right)}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{1}{\left(b\sin\left(x+\frac{\pi}{2}\right)^3+a\right)\tan\left(x+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \; 3709$$

$$-\int \frac{(1-\cos^2(x))\sec^3(x)}{b\cos^3(x)+a}d\cos(x)$$

$$\Big\downarrow \text{2373}$$

$$-\int \left(\frac{\sec^3(x)}{a}-\frac{\sec(x)}{a}+\frac{b(\cos^2(x)-1)}{a\,(b\cos^3(x)+a)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cos(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\cos^2(x)\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}+\frac{\log(\cos(x))}{a}$$

input
```
Int[Tan[x]^3/(a + b*Cos[x]^3),x]
```

output
```
-((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Cos[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3))) + Log[Cos[x]]/a + (b^(2/3)*Log[a^(1/3) + b^(1/3)*Cos[x]])/(3*a
^(5/3)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Cos[x] + b^(2/3)*Cos[x]^2
])/(6*a^(5/3)) - Log[a + b*Cos[x]^3]/(3*a) + Sec[x]^2/(2*a)
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.92.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}a}+i\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{3}a^{5}-27ia^{4}\_Z^{2}-9\_Za^{3}+ia^{2}-ib^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{6ia^{2}\_R}{b}+\frac{2a}{b}\right)\mathrm{e}^{ix}+1\right)\right)-$
default	$\frac{\ln(\cos(x))}{a}+\frac{1}{2a\cos(x)^{2}}-\frac{\left(-\frac{\ln\left(\cos(x)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos^{2}(x)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(x)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(a+b\left(\cos^{3}\right.\right.}{3b}\right.}{a}$

input
```
int(tan(x)^3/(a+b*cos(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2*exp(2*I*x)/(exp(2*I*x)+1)^2/a+I*sum(_R*ln(exp(2*I*x)+(6*I/b*a^2*_R+2/b*a
)*exp(I*x)+1),_R=RootOf(27*_Z^3*a^5-27*I*a^4*_Z^2-9*_Z*a^3+I*a^2-I*b^2))+1
/a*ln(exp(2*I*x)+1)
```

3.92.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 823, normalized size of antiderivative = 5.38

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^
(1/3) + 2/a)*a*cos(x)^2*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2
/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a^2 + b*cos(x) + a) - 12*cos(x)^2*log
(-cos(x)) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a
^5)^(1/3) + 2/a)*a*cos(x)^2 + 3*sqrt(1/3)*a*sqrt(-(((1/2)^(1/3)*(I*sqrt(3)
 + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1
/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4
)/a^2)*cos(x)^2 - 6*cos(x)^2)*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a^2 + 3/2*sqrt(1/3)*a^2*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/
a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a
^5)^(1/3) + 2/a)*a + 4)/a^2) + 2*b*cos(x) - a) - (((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a*cos(x)^2 - 3*sqrt(
1/3)*a*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/
a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5
- (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2)*cos(x)^2 - 6*cos(x)^2)*log(-1/
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
 2/a)*a^2 + 3/2*sqrt(1/3)*a^2*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
 b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2) - 2*...
```

3.92.6 Sympy [F]

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = \int \frac{\tan^3{(x)}}{a+b\cos^3{(x)}}\,dx$$

input
```
integrate(tan(x)**3/(a+b*cos(x)**3),x)
```

output
```
Integral(tan(x)**3/(a + b*cos(x)**3), x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = \frac{\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\cos(x)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2} - \frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(x)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\cos(x)\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log(\cos(x))}{a} + \frac{1}{2a\cos(x)^2}$$

input
```
integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b)^(
1/3) - 2*cos(x))/(a/b)^(1/3))/a^2 - 1/6*(2*(a/b)^(2/3) + 1)*log(cos(x)^2 -
 (a/b)^(1/3)*cos(x) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*((a/b)^(2/3) - 1)
*log((a/b)^(1/3) + cos(x))/(a*(a/b)^(2/3)) + log(cos(x))/a + 1/2/(a*cos(x)
^2)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\cos(x)\right|\right)}{3a^2} - \frac{\log\left(\left|b\cos(x)^3+a\right|\right)}{3a} + \frac{\log(|\cos(x)|)}{a} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\cos(x)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(\cos(x)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2} + \frac{1}{2a\cos(x)^2}$$

input
```
integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="giac")
```

output
```
-1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + cos(x)))/a^2 - 1/3*log(abs(b*c
os(x)^3 + a))/a + log(abs(cos(x)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1
/3*sqrt(3)*((-a/b)^(1/3) + 2*cos(x))/(-a/b)^(1/3))/a^2 + 1/6*(-a*b^2)^(1/3
)*log(cos(x)^2 + (-a/b)^(1/3)*cos(x) + (-a/b)^(2/3))/a^2 + 1/2/(a*cos(x)^2
)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 1281, normalized size of antiderivative = 8.37

$$\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx = \text{Too large to display}$$

input
```
int(tan(x)^3/(a + b*cos(x)^3),x)
```

output
```
(2*tan(x/2)^2)/(a - 2*a*tan(x/2)^2 + a*tan(x/2)^4) + log(tan(x/2)^2 - 1)/a
 + symsum(log((262144*(9*a*b^10 - b^11 - 37*a^2*b^9 + 85*a^3*b^8 - 107*a^4
*b^7 + 43*a^5*b^6 + 73*a^6*b^5 - 121*a^7*b^4 + 72*a^8*b^3 - 16*a^9*b^2))/a
^6 + root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(27*a^
5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + a^2 - b^2, z, k)*((262144*(72*a^5*b^9 - 96*a^6*b^8 + 1428*
a^7*b^7 - 3684*a^8*b^6 + 612*a^9*b^5 + 3972*a^10*b^4 - 2112*a^11*b^3 - 192
*a^12*b^2))/a^6 + root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k
)*(root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*((262144*(518
4*a^10*b^6 - 3024*a^9*b^7 + 1728*a^11*b^5 - 6048*a^12*b^4 + 1296*a^13*b^3
+ 864*a^14*b^2))/a^6 - root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2,
 z, k)*((262144*(1296*a^10*b^7 - 3888*a^11*b^6 + 2592*a^12*b^5 + 2592*a^13
*b^4 - 3888*a^14*b^3 + 1296*a^15*b^2))/a^6 - (262144*tan(x/2)^2*(1296*a^10
*b^7 - 11016*a^11*b^6 + 27216*a^12*b^5 - 28512*a^13*b^4 + 12960*a^14*b^3 -
 1944*a^15*b^2))/a^6) + (262144*tan(x/2)^2*(4104*a^9*b^7 - 16740*a^10*b^6
+ 18468*a^11*b^5 - 1836*a^12*b^4 - 5292*a^13*b^3 + 1296*a^14*b^2))/a^6) +
(262144*(288*a^7*b^8 - 1836*a^8*b^7 - 1692*a^9*b^6 + 6084*a^10*b^5 + 108*a
^11*b^4 - 4248*a^12*b^3 + 1296*a^13*b^2))/a^6 + (262144*tan(x/2)^2*(4392*a
^8*b^7 - 360*a^7*b^8 + 3366*a^9*b^6 - 29934*a^10*b^5 + 35946*a^11*b^4 - 15
354*a^12*b^3 + 1944*a^13*b^2))/a^6) - (262144*tan(x/2)^2*(72*a^5*b^9 - ...
```

3.93 $\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx$

3.93.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx=\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+b\cos^3(x)}$$

output
```
2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))*a^(1/2)-2/3*(a+b*cos(x)^3)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx=\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+b\cos^3(x)}$$

input
```
Integrate[Sqrt[a + b*Cos[x]^3]*Tan[x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/3 - (2*Sqrt[a + b*Cos[x]
^3])/3
```

3.93.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 25, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a+b\cos^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^3}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^3+a}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3709}$$

$$-\int \sqrt{b\cos^3(x)+a}\sec(x)d\cos(x)$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \sqrt{b\cos^3(x)+a}\sec(x)d\cos^3(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-a\int \frac{\sec(x)}{\sqrt{b\cos^3(x)+a}}d\cos^3(x)-2\sqrt{a+b\cos^3(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{2a\int \frac{1}{\frac{\cos^6(x)}{b}-\frac{a}{b}}d\sqrt{b\cos^3(x)+a}}{b}-2\sqrt{a+b\cos^3(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-2\sqrt{a+b\cos^3(x)}\right)$$

input
```
Int[Sqrt[a + b*Cos[x]^3]*Tan[x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]] - 2*Sqrt[a + b*Cos[x]^3])
/3
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.93.4 Maple [F]

$$\int \sqrt{a+b\left(\cos^{3}\left(x\right)\right)}\,\tan\left(x\right)dx$$

input
```
int((a+b*cos(x)^3)^(1/2)*tan(x),x)
```

output
```
int((a+b*cos(x)^3)^(1/2)*tan(x),x)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.73

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx = \left[\frac{1}{6}\sqrt{a}\log\left(-\frac{b^2\cos\left(x\right)^6+8\,ab\cos\left(x\right)^3+4\left(b\cos\left(x\right)^3+2\,a\right)\sqrt{b\cos\left(x\right)^3+a}\sqrt{a}+8\,a^2}{\cos\left(x\right)^6}\right) - \frac{2}{3}\sqrt{b\cos\left(x\right)^3+a}, -\frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\sqrt{b\cos\left(x\right)^3+a}\sqrt{-a}}{b\cos\left(x\right)^3+2\,a}\right)-\frac{2}{3}\sqrt{b\cos\left(x\right)^3+a}\right]$$

input
```
integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
[1/6*sqrt(a)*log(-(b^2*cos(x)^6 + 8*a*b*cos(x)^3 + 4*(b*cos(x)^3 + 2*a)*sq
rt(b*cos(x)^3 + a)*sqrt(a) + 8*a^2)/cos(x)^6) - 2/3*sqrt(b*cos(x)^3 + a),
-1/3*sqrt(-a)*arctan(2*sqrt(b*cos(x)^3 + a)*sqrt(-a)/(b*cos(x)^3 + 2*a)) -
 2/3*sqrt(b*cos(x)^3 + a)]
```

3.93.6 Sympy [F]

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx = \int \sqrt{a+b\cos^3(x)}\tan(x)\,dx$$

input
```
integrate((a+b*cos(x)**3)**(1/2)*tan(x),x)
```

output
```
Integral(sqrt(a + b*cos(x)**3)*tan(x), x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx = -\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{b\cos(x)^3+a}-\sqrt{a}}{\sqrt{b\cos(x)^3+a}+\sqrt{a}}\right) - \frac{2}{3}\sqrt{b\cos(x)^3+a}$$

input
```
integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
-1/3*sqrt(a)*log((sqrt(b*cos(x)^3 + a) - sqrt(a))/(sqrt(b*cos(x)^3 + a) +
sqrt(a))) - 2/3*sqrt(b*cos(x)^3 + a)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx = -\frac{2\,a\arctan\left(\frac{\sqrt{b\cos(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} - \frac{2}{3}\sqrt{b\cos(x)^3+a}$$

input
```
integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="giac")
```

output
```
-2/3*a*arctan(sqrt(b*cos(x)^3 + a)/sqrt(-a))/sqrt(-a) - 2/3*sqrt(b*cos(x)^
3 + a)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx = \int \tan(x)\,\sqrt{b\cos\left(x\right)^3+a}\,dx$$

input
```
int(tan(x)*(a + b*cos(x)^3)^(1/2),x)
```

output
```
int(tan(x)*(a + b*cos(x)^3)^(1/2), x)
```

3.94 $\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx$

3.94.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[Tan[x]/Sqrt[a + b*Cos[x]^3],x]
```

output
```
(2*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.94.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^3}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^3+a}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3709}$$

$$-\int \frac{\sec(x)}{\sqrt{b\cos^3(x)+a}}d\cos(x)$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{1}{3}\int \frac{\sec(x)}{\sqrt{b\cos^3(x)+a}}d\cos^3(x)$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2\int \frac{1}{\frac{\cos^6(x)}{b}-\frac{a}{b}}d\sqrt{b\cos^3(x)+a}}{3b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[Tan[x]/Sqrt[a + b*Cos[x]^3],x]
```

output
```
(2*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.94.4 Maple [F]

$$\int \frac{\tan (x)}{\sqrt{a+b\left(\cos ^{3}(x)\right)}}dx$$

input
```
int(tan(x)/(a+b*cos(x)^3)^(1/2),x)
```

output
```
int(tan(x)/(a+b*cos(x)^3)^(1/2),x)
```

3.94.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   fail
ed of mode Union(SparseUnivariatePolynomial(Expression(Complex(Integer))),
failed) cannot be coerced to mode SparseUnivariatePolynomial(Expression(Co
mplex(Int
```

3.94.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx = \int \frac{\tan\left(x\right)}{\sqrt{a+b\cos^3\left(x\right)}}\,dx$$

input
```
integrate(tan(x)/(a+b*cos(x)**3)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a + b*cos(x)**3), x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx = -\frac{\log\left(\frac{\sqrt{b\cos(x)^3+a}-\sqrt{a}}{\sqrt{b\cos(x)^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*log((sqrt(b*cos(x)^3 + a) - sqrt(a))/(sqrt(b*cos(x)^3 + a) + sqrt(a))
)/sqrt(a)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{b\cos(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

input
```
integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="giac")
```

output
```
-2/3*arctan(sqrt(b*cos(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{b\cos{(x)}^3+a}}\,dx$$

input
```
int(tan(x)/(a + b*cos(x)^3)^(1/2),x)
```

output
```
int(tan(x)/(a + b*cos(x)^3)^(1/2), x)
```

3.95 $\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx$

3.95.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx=\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+b\cos^4(x)}$$

output
```
1/2*arctanh((a+b*cos(x)^4)^(1/2)/a^(1/2))*a^(1/2)-1/2*(a+b*cos(x)^4)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx=\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+b\cos^4(x)}$$

input
```
Integrate[Sqrt[a + b*Cos[x]^4]*Tan[x],x]
```

output
```
(Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]])/2 - Sqrt[a + b*Cos[x]^4]/2
```

3.95.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 25, 3708, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a+b\cos^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^4}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^4+a}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3708}$$

$$-\frac{1}{2}\int \sqrt{b\cos^4(x)+a}\sec^2(x)d\cos^2(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{4}\int \sqrt{b\cos^4(x)+a}\sec^2(x)d\cos^4(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\int \frac{\sec^2(x)}{\sqrt{b\cos^4(x)+a}}d\cos^4(x)-2\sqrt{a+b\cos^4(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{2a\int \frac{1}{\frac{\sqrt{b\cos^4(x)+a}}{b}-\frac{a}{b}}d\sqrt{b\cos^4(x)+a}}{b}-2\sqrt{a+b\cos^4(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-2\sqrt{a+b\cos^4(x)}\right)$$

input
```
Int[Sqrt[a + b*Cos[x]^4]*Tan[x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]] - 2*Sqrt[a + b*Cos[x]^4])
/4
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.95.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\sqrt{a+b\left(\cos^4(x)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^4(x)\right)}}{\cos(x)^2}\right)}{2}$	44

input
```
int((a+b*cos(x)^4)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a+b*cos(x)^4)^(1/2)+1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^4)^(1/
2))/cos(x)^2)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.00

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx = \left[\frac{1}{4}\sqrt{a}\log\left(\frac{b\cos(x)^4+2\sqrt{b\cos(x)^4+a}\sqrt{a}+2a}{\cos(x)^4}\right) - \frac{1}{2}\sqrt{b\cos(x)^4+a}, -\frac{1}{2}\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^4+a}\sqrt{-a}}{a}\right) - \frac{1}{2}\sqrt{b\cos(x)^4+a}\right]$$

input
```
integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log((b*cos(x)^4 + 2*sqrt(b*cos(x)^4 + a)*sqrt(a) + 2*a)/cos(x
)^4) - 1/2*sqrt(b*cos(x)^4 + a), -1/2*sqrt(-a)*arctan(sqrt(b*cos(x)^4 + a)
*sqrt(-a)/a) - 1/2*sqrt(b*cos(x)^4 + a)]
```

3.95.6 Sympy [F]

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx = \int \sqrt{a+b\cos^4{(x)}}\tan{(x)}\,dx$$

input
```
integrate((a+b*cos(x)**4)**(1/2)*tan(x),x)
```

output
```
Integral(sqrt(a + b*cos(x)**4)*tan(x), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx = \frac{1}{2}\sqrt{a}\,\text{arsinh}\left(-\frac{a}{\sqrt{ab}\left(\sin{(x)}^2-1\right)}\right) - \frac{1}{2}\sqrt{b\sin{(x)}^4-2\,b\sin{(x)}^2+a+b}$$

input
```
integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*arcsinh(-a/(sqrt(a*b)*(sin(x)^2 - 1))) - 1/2*sqrt(b*sin(x)^4 -
 2*b*sin(x)^2 + a + b)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx = -\frac{a\arctan\left(\frac{\sqrt{b\cos{(x)}^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} - \frac{1}{2}\sqrt{b\cos{(x)}^4+a}$$

input
```
integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

output
```
-1/2*a*arctan(sqrt(b*cos(x)^4 + a)/sqrt(-a))/sqrt(-a) - 1/2*sqrt(b*cos(x)^
4 + a)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos^4(x)}\tan(x)\,dx=\int \tan(x)\,\sqrt{b\cos{(x)}^4+a}\,dx$$

input
```
int(tan(x)*(a + b*cos(x)^4)^(1/2),x)
```

output
```
int(tan(x)*(a + b*cos(x)^4)^(1/2), x)
```

3.96 $\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx$

3.96.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
1/2*arctanh((a+b*cos(x)^4)^(1/2)/a^(1/2))/a^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[Tan[x]/Sqrt[a + b*Cos[x]^4],x]
```

output
```
ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3708, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^4}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^4+a}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3708}$$

$$-\frac{1}{2}\int \frac{\sec^2(x)}{\sqrt{b\cos^4(x)+a}}d\cos^2(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{4}\int \frac{\sec^2(x)}{\sqrt{b\cos^4(x)+a}}d\cos^4(x)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{\sqrt{b\cos^4(x)+a}}{b}-\frac{a}{b}}d\sqrt{b\cos^4(x)+a}}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Int[Tan[x]/Sqrt[a + b*Cos[x]^4],x]
```

output
```
ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.96.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^{4}(x)\right)}}{\cos(x)^{2}}\right)}{2\sqrt{a}}$	31

input
```
int(tan(x)/(a+b*cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^4)^(1/2))/cos(x)^2)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = \left[\frac{\log\left(\frac{b\cos(x)^4+2\sqrt{b\cos(x)^4+a}\sqrt{a}+2a}{\cos(x)^4}\right)}{4\sqrt{a}}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^4+a}\sqrt{-a}}{a}\right)}{2a}\right]$$

input
```
integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((b*cos(x)^4 + 2*sqrt(b*cos(x)^4 + a)*sqrt(a) + 2*a)/cos(x)^4)/sqr
t(a), -1/2*sqrt(-a)*arctan(sqrt(b*cos(x)^4 + a)*sqrt(-a)/a)/a]
```

3.96.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx$$

input
```
integrate(tan(x)/(a+b*cos(x)**4)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a + b*cos(x)**4), x)
```

3.96.7 Maxima [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{b\cos(x)^4+a}}\,dx$$

input
```
integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/sqrt(b*cos(x)^4 + a), x)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\cos(x)^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

input
```
integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/2*arctan(sqrt(b*cos(x)^4 + a)/sqrt(-a))/sqrt(-a)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{b\cos(x)^4+a}}\,dx$$

input
```
int(tan(x)/(a + b*cos(x)^4)^(1/2),x)
```

output
```
int(tan(x)/(a + b*cos(x)^4)^(1/2), x)
```

3.97 $\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx$

3.97.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx=\frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{a+b\cos^n(x)}}{n}$$

output
```
2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n-2*(a+b*cos(x)^n)^(1/2)/n
```

3.97.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx=-\frac{-2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cos^n(x)}}{n}$$

input
```
Integrate[Sqrt[a + b*Cos[x]^n]*Tan[x],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cos[x]^
n])/n)
```

3.97.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 25, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a+b\cos^n(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^n}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^n+a}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3709$$

$$-\int \sqrt{b\cos^n(x)+a}\sec(x)d\cos(x)$$

$$\Big\downarrow \; 798$$

$$-\frac{\int \sqrt{b\cos^n(x)+a}\sec(x)d\cos^n(x)}{n}$$

$$\Big\downarrow \; 60$$

$$-\frac{a\int \frac{\sec(x)}{\sqrt{b\cos^n(x)+a}}d\cos^n(x)+2\sqrt{a+b\cos^n(x)}}{n}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{2a\int \frac{1}{\frac{\cos^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\cos^n(x)+a}}{b}+2\sqrt{a+b\cos^n(x)}}{n}$$

$$\Big\downarrow \; 221$$

$$-\frac{2\sqrt{a+b\cos^n(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[a + b*Cos[x]^n]*Tan[x],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cos[x]^
n])/n)
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\sqrt{a+b(\cos^n(x))}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n}$	39
default	$-\frac{2\sqrt{a+b(\cos^n(x))}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n}$	39

input
```
int((a+b*cos(x)^n)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/n*(2*(a+b*cos(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2
)))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx = \left[\frac{\sqrt{a}\log\left(\frac{b\cos(x)^n+2\sqrt{b\cos(x)^n+a}\sqrt{a}+2\,a}{\cos(x)^n}\right)-2\sqrt{b\cos(x)^n+a}}{n}, -\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^n+a}\sqrt{-a}}{a}\right)+\sqrt{b\cos(x)^n+a}\right)}{n}\right]$$

input
```
integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*cos(x)^n + 2*sqrt(b*cos(x)^n + a)*sqrt(a) + 2*a)/cos(x)^n
) - 2*sqrt(b*cos(x)^n + a))/n, -2*(sqrt(-a)*arctan(sqrt(b*cos(x)^n + a)*sq
rt(-a)/a) + sqrt(b*cos(x)^n + a))/n]
```

3.97.6 Sympy [F]

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx = \int \sqrt{a+b\cos^n(x)}\tan(x)\,dx$$

input
```
integrate((a+b*cos(x)**n)**(1/2)*tan(x),x)
```

output
```
Integral(sqrt(a + b*cos(x)**n)*tan(x), x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx = -\frac{\sqrt{a}\log\left(\frac{\sqrt{b\cos(x)^n+a}-\sqrt{a}}{\sqrt{b\cos(x)^n+a}+\sqrt{a}}\right)}{n} - \frac{2\sqrt{b\cos(x)^n+a}}{n}$$

input
```
integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
-sqrt(a)*log((sqrt(b*cos(x)^n + a) - sqrt(a))/(sqrt(b*cos(x)^n + a) + sqrt
(a)))/n - 2*sqrt(b*cos(x)^n + a)/n
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx = -\frac{2\left(\frac{ab\arctan\left(\frac{\sqrt{b\cos(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\cos(x)^n+a}\,b\right)}{bn}$$

input
```
integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="giac")
```

output
```
-2*(a*b*arctan(sqrt(b*cos(x)^n + a)/sqrt(-a))/sqrt(-a) + sqrt(b*cos(x)^n +
 a)*b)/(b*n)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos^n(x)}\tan(x)\,dx=\int \tan(x)\,\sqrt{a+b\cos\left(x\right)^n}\,dx$$

input
```
int(tan(x)*(a + b*cos(x)^n)^(1/2),x)
```

output
```
int(tan(x)*(a + b*cos(x)^n)^(1/2), x)
```

3.98 $\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx$

3.98.1 Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[Tan[x]/Sqrt[a + b*Cos[x]^n],x]
```

output
```
(2*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.98.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^n}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^n+a}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3709$$

$$-\int \frac{\sec(x)}{\sqrt{b\cos^n(x)+a}}d\cos(x)$$

$$\Big\downarrow \; 798$$

$$-\frac{\int \frac{\sec(x)}{\sqrt{b\cos^n(x)+a}}d\cos^n(x)}{n}$$

$$\Big\downarrow \; 73$$

$$-\frac{2\int \frac{1}{\frac{\cos^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\cos^n(x)+a}}{bn}$$

$$\Big\downarrow \; 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[Tan[x]/Sqrt[a + b*Cos[x]^n],x]
```

output
```
(2*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24
default	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24

input
```
int(tan(x)/(a+b*cos(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.55

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = \left[\frac{\log\left(\frac{b\cos(x)^n+2\sqrt{b\cos(x)^n+a}\sqrt{a}+2a}{\cos(x)^n}\right)}{\sqrt{a}n}, -\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

input
```
integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*cos(x)^n + 2*sqrt(b*cos(x)^n + a)*sqrt(a) + 2*a)/cos(x)^n)/(sqrt(a
)*n), -2*sqrt(-a)*arctan(sqrt(b*cos(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

3.98.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx$$

input
```
integrate(tan(x)/(a+b*cos(x)**n)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a + b*cos(x)**n), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = -\frac{\log\left(\frac{\sqrt{b\cos(x)^n+a}-\sqrt{a}}{\sqrt{b\cos(x)^n+a}+\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="maxima")
```

output
```
-log((sqrt(b*cos(x)^n + a) - sqrt(a))/(sqrt(b*cos(x)^n + a) + sqrt(a)))/(s
qrt(a)*n)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{b\cos(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}n}$$

input
```
integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(b*cos(x)^n + a)/sqrt(-a))/(sqrt(-a)*n)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a+b\cos(x)^n}}\,dx$$

input
```
int(tan(x)/(a + b*cos(x)^n)^(1/2),x)
```

output
```
int(tan(x)/(a + b*cos(x)^n)^(1/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```